PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

On the Determination of the Specific Resistance of Mercury in Absolute Measure

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PHILOSOPHICAL TRANSACTIONS.

I. *On the Determination of the Specific Resistance of Mercury in Absolute Measure.*

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[PLATES 1–3.]

§ 1. "ON the whole I am of opinion that if it is desirable at the present time to construct apparatus on the most favourable scale, so as to reach the highest attainable accuracy, the modification of LORENZ's method last described is the one that offers the best prospect of success. Before this is done however, it appears to me important that the value now three times obtained in the Cavendish Laboratory by distinct methods should be approximately verified (or disproved) by other physicists. To distinguish between this value and those obtained for instance by KOHLRAUSCH, by LORENZ, or by the First B.A. Committee, should not require the construction of unusually costly apparatus. Until the larger question is disposed of it appears premature to discuss the details of arrangements from which the highest degree of precision is to be expected."

The above passage, which concludes a paper communicated by Lord RAYLEIGH to the 'Philosophical Magazine,'* a little before the Electrical Congress at Paris, at which the legal ohm was defined to be the resistance of a column of mercury of 1 sq. mm. section and 1060 mm. long, seems not to have met with adequate response in this country. So far as experiments in English Laboratories are concerned the

* 'Phil. Mag.,' November, 1882, "Comparison of Methods for the Determination of Resistances in Absolute Measure."

determination of the ohm remains where Lord RAYLEIGH left it, except for the contribution made by GLAZEBROOK and FITZPATRICK in their re-measurement of the Specific Resistance of Mercury in terms of the B.A. Unit, which is one of the elements in the determination of the Specific Resistance of Mercury in Absolute Measure by Lord RAYLEIGH's adaptation of LORENZ's method.

The measurements to be described in this paper were undertaken in the hope of paving the way for the final determination referred to by LORD RAYLEIGH ; and with a view of submitting to the test of experiment certain modifications of LORENZ's method, which had occurred to the author as likely to lead to increased accuracy and certainty. Therefore, though the results are given it is asked that the work should be regarded rather as justifying a suggested method of measurement than as pretending to be a satisfactory or final determination of the physical constant sought.

Probably the experiments are sufficient to make good the statement that, if the apparatus be constructed on a scale a little larger and with a certain perfecting of detail, a single set of observations will give a result accurate to 1 part in 10,000 ; and the author is of opinion that an even greater accuracy may be obtained by the method if due regard be paid to the maintenance of definite temperatures in all parts of the apparatus.

§ 2. In LORENZ's method a metallic disc is made to rotate in the mean plane of a co-axial standard coil. Wires touching the centre and circumference of the disc are led to the ends of the resistance to be measured; and the same current is passed through this resistance and the standard coil. The connections being rightly made we may by varying either the rate of rotation of the disc or the resistance to be measured, so arrange matters as to have no change of current in the circuit of the disc and wires joining it to the ends of the resistance, when the direction of the current through the resistance and the standard coil is changed. When this arrangement is effected there is a balance between the electromotive force due to the motion of the disc in the magnetic field of the current in the standard coil, and the difference of potential at the ends of the resistance, due to the current traversing it. If this adjustment be made we will say that the apparatus is in an equilibrium position.

If M = the coefficient of mutual induction of the standard coil and the circumference of the disc,
n = the rate of rotation of the disc (number of revolutions per second),
R = the resistance,
γ = the current through the standard coil and the resistance,

then in an equilibrium position

$$Mn\gamma = R\gamma \quad \text{or} \quad R = Mn.$$

The variations in the method introduced in the present investigation are mainly the following :—

(i.) The use of a standard coil with one layer of wire on it.

(ii.) The elimination by a system of differential measurements of the errors that have so far attended the use of a mercury column as the measured resistance.

§ 3. *The Standard Coil.*—The coil C (Plate 1, figs. 1 and 2), is of double silk covered copper wire of about ·02 inch diameter, resting in a screw thread of ·025 inch pitch, cut on a hollow cylinder of brass of approximately 10 inches and 10·5 inches internal and external radius. The axial length of the cylinder is something over five inches and 185 turns of wire are wound on it. Precise details of its dimensions are given later. The lathe on which the cylinder was turned was made for the College Workshop by Sir J. Whitworth and Co. with special care, and it is an exceptionally massive and accurate tool.

The brass cylinder has three lugs L, L, L, by which it was bolted to the face plate of the lathe, the outside faces of the lugs having been carefully scraped to fit the plate. In use the cylinder is bolted in precisely similar fashion to a strong triangular brass frame, supported on three levelling screws (as shown in Plate 1, figs. 1 and 2); so that it is used in a position as like as possible to that in which it was turned.

The first operation was to fit the lugs roughly to the face plate by filing, and a roughing cut was taken over the entire surface of the cylinder, so as to get rid of as much as possible of the internal strain due to unequal cooling of the casting. It was then unbolted; the lugs were carefully scraped to fit the face plate, and the final turning proceeded with. Very fine cuts were taken in finishing, and it was hoped by these precautions that we should secure a true right circular cylinder. Nevertheless our subsequent measurements show that the section of the cylinder at right angles to its axis is slightly oval. It appears, indeed, to be a task of no ordinary difficulty to turn a cylinder of large size that shall remain true to the thousandth of an inch. The screw thread of 40 turns to the inch was next cut on the cylinder to a depth of about ·012 inch.

It is necessary for purposes of adjustment to know the mean plane of the screw thread. The next operation was, therefore, directed to find this plane, and to mark the circle of its intersection with the inner surface of the cylinder.

A microscope and a **V**-tool were fixed to the slide rest, the microscope being at right angles to the lathe bed, and the tool being adjusted so that the image of its point was at the central mark of the graduated glass plate in the eye-piece of the microscope. The microscope and tool were placed in such a position that when the slide rest moved forward the microscope remained outside and the tool passed inside the cylinder. A wheel of 9·75 inches radius divided into 360 parts on its circumference was attached to the guide screw, which is of pitch ·25 inch. A tenth of a division on the wheel could be estimated with tolerable accuracy.

A generating line was then drawn on the outside of the cylinder, and the microscope was focussed so as to bring the ridges of the screw thread on this line into the field of view. Next the guide screw was turned so as to bring first the right hand and then the left hand ridge of the first thread on to the central mark in the field of view. The mean of the readings of the wheel in these two positions was taken to be the reading for the centre of the hollow.

The wheel and guide screw were then turned until first the right hand and then the left hand ridges of the last thread on the cylinder were in like manner brought on to the central mark, the mean of these readings being again taken as the reading for the centre of the hollow. The number of turns and fractions of a turn made in order to bring the microscope from the first position to the second position gives the axial length between the first and last hollows on the generating line. By adding half this number to the reading for the first hollow, the reading for the mean plane was obtained. The wheel and guide screw were then moved to this position, and a circle was cut inside the cylinder. This circular cut is used in adjusting the mean plane of the coil to the plane of the disc.

Small holes were then radially bored through the brass of the cylinder from the points where the generating line along which the measurements had been taken intersected the first and last hollows, and these holes were bushed with well paraffined ebonite.

The next step was to wind the wire in the screw thread. The wire was led from the bobbin over a pulley fixed some feet above it, under a movable pulley with a weight attached, over another fixed pulley, and finally under a pulley fixed to the slide rest which delivered it into the screw thread on the brass cylinder. The object of the movable pulley with the weight attached was to keep the tension in the wire uniform during the winding.

One end of the wire was passed through the left hand small hole bushed with ebonite above referred to, and secured; the guide screw was then thrown in gear and the cylinder as it rotated drew the wire uniformly into place. When the winding was complete the wire was clamped about three inches from the right hand hole until the cut end was passed through and drawn tight. Melted paraffin was then run into the holes in the bushes to secure the ends of the wire. Blocks of ebonite were afterwards attached to the brass cylinder at the ends of the generating line before-mentioned; and binding screws were fixed in them, to which the ends of the wire were soldered.

Tests, the details of which are given in § 21, show that the coil is well insulated.

The advantage in constructing a standard coil with a single layer of wire is that every part of it is visible, and that nothing is done to alter the position of the wire after measurements are made. If a coil consists of many layers, it is not quite easy to say where the lower layers place themselves after measurement under the pressure of the superincumbent layers.

Lord RAYLEIGH* found, in the case of certain coils used by him, that the mean radius calculated from measurements made in winding was greater by 1 part in 2000 than the mean radius calculated from measurements made in unwinding, a result clearly due to the displacement of the lower layers ('Phil. Trans.,' 1882) (*cf.* GLAZE-

* 'Phil. Trans.,' 1882, pp. 673–4.

BROOK and SARGANT, 'Phil. Trans.,' 1883, p. 267). We should therefore expect that if the mean radius of a coil of many layers is calculated from measurements made in winding it will be reckoned too large.

In the face of this, it is interesting to note that in the measurements made by Lord RAYLEIGH and Mrs. SIDGWICK by the method of LORENZ, the series in which the coils are so placed as to make the induction coefficient "nearly independent of a knowledge of the mean radius," gives a result rather larger than the other series, which is what might be expected if the mean radius is overestimated. But the smallness of the variation (about 1 part in 10,000) seems to show that the error of mean radius must be very small for the pair of coils (wound by Professor CHRYSTAL) used by Lord RAYLEIGH in these experiments.

In general, we should expect, as far as this cause goes, that values of the specific resistance in determining which coils of many ayers are used, would come out too low, since an overestimation of the mean radius means an underestimation of the coefficient of mutual induction.

The apparent disadvantage in a coil of a single layer is that the number of turns will be fewer, and, therefore, the coefficient of mutual induction of the coil and disc smaller than in the case of a coil of many layers. For some purposes this might be a grave objection; but it is not so for the work in hand. Although the coefficient of mutual induction of the coil and disc is only a little more than one-twentieth of that of Lord RAYLEIGH's coils and disc in the first and second series of his measurements by the method of LORENZ, yet it seems ample for the purpose.

In order, however, to get a sufficient number of turns on the coil of a single layer, its axial length is made too long to allow of our calculating the coefficient of mutual induction of the coil and disc by Lord RAYLEIGH's formula of approximation. It, therefore, became necessary to calculate a new formula, which was obtained by the direct integration of the expression $\iint (ds\,ds'/r) \cos \epsilon$ for a circle and coaxial helix, and communicated to the Physical Society (*vide* 'Phil. Mag.,' Jan., 1889). By Lord RAYLEIGH's formula the coefficient in these experiments would be given as $n \times 90{\cdot}0383$, its value by the formula for circle and helix being $n \times 89{\cdot}7717$. The difference amply confirms the need of the new formula for a coil of such axial length as that used.

§ 4. *The Use of a Mercury Column by the Differential Method.*—The smallness of the resistance to be measured has usually been regarded as one of the difficulties of LORENZ's method. If, on the one hand, mercury has been used, there has been the difficulty of calculating the specific resistance from the measured resistance of the mercury filling a glass tube of a large sectional area; while, on the other hand, where solid conductors have been used there has been the difficulty of comparing the resistances measured with ordinary standards. Lord RAYLEIGH, by a shunt method, overcame the latter difficulty. But, notwithstanding the success which attended the use of this method, it has seemed better to the author to revert to LORENZ's use of mercury.

The physical constant to be measured is the specific resistance of mercury. If this can, by direct measurement, be determined with accuracy, it seems inconvenient and undesirable first to determine the resistance of the B.A. unit in absolute measure, and then to determine the specific resistance of mercury in B.A. units. If, consistently with accuracy, the artificial B.A. unit can be dropped out of our experiments as well as out of the result, the measurements being made direct on mercury, the simplicity would seem to be a recommendation.

The difficulty in the way of using mercury in a glass tube presented itself thus: if the wires from the disc (the terminal portions of which may be called the electrodes) are led to the ends of the tube, the equipotential surfaces touched by them are not plane; and if they are let into the tube at some distance from the ends, it is difficult to see how the distance between them is to be determined with the requisite accuracy.

Both these difficulties disappear if, instead of placing the mercury in a glass tube, it is placed in a long trough, and if, instead of measuring the distance between the electrodes, one electrode is kept fixed while measurement is made of the distance moved through by the other between two equilibrium positions corresponding to two different rates of rotation of the disc. The latter measurement it is easy to make with accuracy, for the movable electrode may be rigidly attached to the movable headstock of a Whitworth measuring machine, or some other measuring bank placed parallel to the length of the trough. The two equilibrium positions may be taken near the middle of the trough, so as to avoid danger of curvature in the equipotential surfaces passing through the electrode in its two positions.

Let n_1, n_2 be the rates of rotation of the disc, and let l be the distance between the corresponding equilibrium positions of the movable electrode.

Then

$$\mathrm{M}\,(n_1 - n_2) = \frac{l}{\mathrm{A}}\,\rho,$$

where $\rho =$ the specific resistance of mercury, and $\mathrm{A} =$ the area of section of the mercury column.

But we are met by a new difficulty, the determination of the section of the mercury column. The capillary depression at the sides of the trough would make it a serious task to determine the section by direct measurement to the required degree of accuracy. Fortunately this difficulty may be overcome by a further differential method, viz., by making observations with the mercury at two different heights in the trough.

Let

$b =$ the breadth of the trough,

$h_2 - h_1 =$ the difference of height of the mercury surface in its two positions,

and let

$\mathrm{A} =$ the section of the mercury column when the mercury surface is at the lower position.

Then we have, denoting by dashed letters the new values of the rates of rotation and the distance between the corresponding equilibrium positions,

$$M(n_1 - n_2) = \frac{l}{A}\rho,$$

and

$$M(n_1' - n_2') = \frac{l'}{A + b(h_2 - h_1)}\rho,$$

whence, eliminating A, we have

$$\rho = \frac{Mb(h_2 - h_1)}{\dfrac{l'}{n_1' - n_2'} - \dfrac{l}{n_1 - n_2}}.$$

It is assumed in the above calculation that the sides of the trough are plane, parallel, and vertical; and that the temperature remains constant.

Hence the determination of the specific resistance involves the determination of :—

(i.) Four equilibrium positions, two at each depth, with the rates of rotation of the disc to which they correspond.
(ii.) The breadth of the trough.
(iii.) The difference of level of the mercury surface at the two depths.
(iv.) The coefficient of mutual induction of the coil and disc.

§ 5. The general arrangement of the apparatus used in the experiments to be described is shown in Plate 2, fig. 5.

M. Electromotor.
g. Flexible coupling.
$\beta, \beta, \beta, \beta$. Bearings.
W. Fly-wheel.
Dr. Cylinder, with rows of black and white teeth, for measurement of rate of rotation.
F. Tuning-fork box.
C. Standard coil.
D. Disc.
E. Battery of secondary cells.
m, m. Whitworth measuring machine.
Y. Loose headstock on measuring machine.
Z. Fixed ,, ,, ,,
Tr. Mercury trough.
p_1, p_2. Iron plates to which the battery wires are soldered.
T_1, T_2. Thermometers giving the temperature of the mercury in the trough.
σ. Spherometer.
O. Movable electrode.
P. Fixed electrode.

X, X, X, X. Box surrounding trough and measuring machine.
K_1, K_2, K_3. Keys.
U_1. Coil for adjusting the induction, so as to obtain the required throw of the galvanometer needle when the current is reversed at K_2.
U_2. Coil for adjusting, so that reversal of the battery current shall produce no effect on the galvanometer needle when the galvanometer circuit is broken.
G. Galvanometer.
S. Lamp and scale.

§ 6. *The Mercury Trough.*—The trough used was cut in paraffin wax, and was prepared in the following manner. A strong casting of iron was made with its sides strengthened by outside ribs, as shown in Plate 2, figs. 6, 7, and 8. Its internal dimensions are 51·25 inches in length, by about 3 inches in breadth, by about 4 inches in depth. To fill it with paraffin wax, the casting was placed with its central part over a stove, and BUNSEN burners were inserted under the projecting ends. Paraffin wax, cut in small pieces, was then put in the trough, and as this melted, more wax was added until the trough was quite full of molten paraffin. The fire and burners were then removed, and the whole left to cool slowly. After a solid coat had formed on the surface, a hole was made through it to the liquid paraffin in the interior, and through this hole a further quantity of melted wax was poured during the progress of solidification. This process was continued until the iron case was filled with solid paraffin wax.

The case so filled was then fastened to the bed of the lathe with its length parallel to the run of the bed and the free paraffin surface upwards. A cutter attached to the slide rest rotating about 2,000 turns per minute round a vertical axis was used to cut out the required channel in the paraffin wax, the channel being approximately 43·5 inches long by 1·5 inches broad by 3 inches deep. When the channel was cut for the first time a certain number of air holes were found in the base and lateral faces. Experiments on a smaller trough had, however, shown that these might be satisfactorily removed by depositing paraffin wax in a thin layer on the sides and base of the channel, and re-cutting. This was successfully accomplished for the large trough, and a uniform surface was secured. The channel was finished by carrying a scraper from end to end which took a very thin cut off both faces and base. The result of the scraping was a very smooth and highly finished surface.

It was thought that the breadth of the trough might be obtained by measuring the width of the scraper. But they were not found to be equal when the breadth of the trough was subsequently measured. The difference amounts to ·006 or ·007 inch, the width of the scraper being the greater. This difference may be due in part to the fact that the measurement of the trough was made at a different temperature (about 15°·5 C.) to that at which it was cut. Unfortunately the latter temperature was not registered.

This method of obtaining a channel for the mercury was adopted in the hope that so cut in the lathe, it might present a breadth constant to about the ten-thousandth of an inch. This hope was not realised. The results of calibrating the trough over the ten inches of it, used in the course of the experiments, are given in § 28. Probably more satisfactory results might be obtained by building up the trough of worked glass or scraped marble.

The objections to the use of paraffin wax are :—

(i.) The softness of the wax makes it necessary to proceed with great care in measuring its breadth lest damage should be done to the surface. This difficulty is satisfactorily surmounted by the form of internal callipers adopted in the calibration.

(ii.) The coefficient of expansion of paraffin wax is very large. It was found by making measurements with the Whitworth Measuring Machine on a bar of paraffin wax, 12 inches long, that the linear coefficient of expansion is as much as ·0003. Distortion would therefore be the result of varying the temperature, since the wax is enclosed in an iron casing. The danger of inaccuracy due to this cause was met by measuring the breadth of the trough at a temperature as nearly as possible the same as that at which the electrical observations were made, care being taken to maintain the trough at a uniform temperature during the observations. As will be seen from Table VI. the extreme temperature readings do not differ from one another by more than ·5° C. But, notwithstanding this care, the author is of opinion that the difference in the results of different sets of observations depends in the main upon variations in the trough due to varying temperature.

§ 7. The disc, axle, and bearings are of phosphor bronze. The details are shown in Plate 1, figs. 1, 2, 3, 4.

The ratio of the radii of disc and coil is ·61635, very nearly that used in the measurements of Lord Rayleigh.

The disc is insulated from the axle by well paraffined ebonite (Plate 1, fig. 3); otherwise there would be a short-circuiting of part of the radius of the disc through the bearings and bed.

It was at first intended to drive the disc by rope gearing from the electromotor, but in the course of preliminary experiments, though the rope was 40 yards long, and the joining of the ends took place over a length of quite six feet, there was a sudden variation in speed, visible at the time-measuring tuning fork, and producing a distinct movement of the galvanometer needle, whenever the joint passed over the pulley. The rope gearing was therefore abandoned, and the motor coupled direct, as in the figure. Unfortunately, by an oversight, the part of the axle, $\xi\,\xi$ (Plate 2, fig. 5), joining the motor to the previously prepared part of the apparatus was made of steel instead of phosphor bronze, and the presence of the steel has some slight effect (about 4·3 parts in 10,000) on the coefficient of mutual induction of the coil and disc. No permanent magnetic field in the steel affects the result; but the greater permeability of the steel increases the coefficient of mutual induction.

The disc was ground true in place by means of a small emery wheel driven by an electromotor, the disc at the same time being rotated.

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§ 8. The brushes are of phosphor bronze wire, of diameter ·116 inch. The brush resting on the circumference of the disc at first gave great trouble. It was found impossible with an ordinary brush made of a number of layers of thin phosphor bronze sheet, and controlled by a spring, to obtain sufficiently constant readings at the galvanometer, however well the circumference of the disc and the brush were initially amalgamated. Trials were made in the hope of improvement with amalgamated copper and amalgamated lead, and with the substitution of a dead weight pressure for the spring. But no satisfactory result could be obtained.

It was noticed, however, that after amalgamation the readings were fairly steady for a short interval; and it seemed likely that if mercury could be continuously supplied to the surface of contact between the brush and the disc, the electromotive force at their contact would be rendered much more constant. This led on to the idea of a brush consisting of a single wire, perforated by a channel, through which a constant flow of mercury might be maintained from a cistern of adjustable height and a brush of this description was finally adopted, the details of which are shown at at Q, Plate 1, figs. 3 and 4.

A piece of copper, an inch long and ·375 inch square, has a hole, ·25 inch diameter, drilled to a depth of ·75 inch, and at right angles to this a second hole of sectional area slightly larger than that of the phosphor bronze wire brush. In the wire brush a channel ·03 inch diameter is drilled to the depth of about an inch, and the wire is brazed into the copper piece, as shown in the drawings. A small hole is then made with a fine pointed drill, connecting the cavity in the copper piece with the channel in the wire. The brush being in place, an india-rubber tube is passed over the free end of the copper piece, which carries to the brush a constant supply of mercury from the cistern. When the apparatus is in use, the height of the cistern is adjusted so that a continuous stream of mercury is dragged out of the brush by the rotating disc.

The extent to which this new form of brush meets the difficulty of varying electromotive force at the contact is best estimated by the actual galvanometer readings taken in the course of the experiments which are detailed in Tables I.–V.* It certainly presents very great advantages over any other form of brush tried.

The central brush Q′ is made in similar fashion, but it is not necessary to have so rapid a flow of mercury. It is sufficient to make the cavity in the copper piece of the mercury cistern and to replenish it at intervals. The details of its position relatively to the disc are shown in fig. 3. It enters the disc to a depth of ·06 inch.

§ 9. The motor used is a Siemens Dynamo machine, to which a steel fly-wheel has been fitted, as shown at W in Plate 2, fig. 5, the current being obtained from secondary cells. The current passes through resistance coils, and may also be varied continuously through a small range by a slide resistance of platinoid wire after the larger adjustment has been made. A shunt worked by a lever provides means of taking out or putting in a small resistance suddenly, so as to allow the observer taking note of the speed to

* Of these tables only Table I. is printed. The others are very similar.

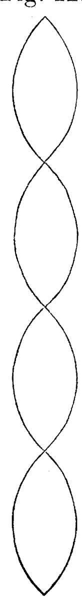
counteract small variations due to alteration in the lubrication of the bearings and the friction of the brushes on the commutator of the motor. The bearings $\beta, \beta, \beta, \beta$, have a length about six times their diameter, and all the bearings are fitted with sight feed lubricators.

The proper lubrication of the bearings seems to be of the greatest importance. In making any final determination it would be necessary to obtain a more constant rate of rotation than has yet been realised, and to maintain it, as far as possible, without interference from the observer. A better result may, perhaps, be achieved if a forced system of lubrication is adopted. The motor brushes would also need further attention.

§ 10. The measurement of the rate of rotation is made with the help of a tuning fork by the method of MACLEOD and CLARKE ('Phil. Trans.,' 1880). At Dr, Plate 2, fig. 5, there is attached to the axle a concentric brass cylinder (made in two pieces, subsequently bolted together round the axle), on the blackened surface of which are painted eleven rows of white teeth, the teeth in any row being equidistant, and the number of teeth in the rows being 7, 8, 9, 10, 11, 12, 13, 15, 17, 19, 21, respectively. In front of this cylinder, or drum, is placed a Koenig tuning fork of pitch approximately 64, the prongs of which have screwed into their ends pieces of brass with narrow central slits about an inch long, parallel to the length of the fork. At the centre of each slit a cross-wire is placed at right angles to its length. The fork is enclosed in a box and a thermometer divided to tenths of a degree is placed between the prongs. The tuning fork is vertical, so that the displacement of any point of a slit is at right angles to that of a tooth.

Fig. 11.

Fig. 12.



When the disc is rotating and the fork in vibration, for an observer looking through the fork slit at the drum, any point of the slit is illuminated, while a white space is passing the line joining the observer's eye to the point of the slit looked at, and unilluminated while a dark space is passing that line. If the interval between the passage of successive white teeth is equal to the periodic time of the fork, any point of the slit looked at will appear fixed in one position, but the position will be different for each point, and the slit as a whole will be seen as an undulating line (fig. 11), stationary if the synchronism is exact, and if not varying so as to give the appearance of a wave moving one way or the other according as the interval between the passage of two teeth is less or greater than the periodic time of the fork.

In practice it is better, however, to adjust so that the interval between the passage of two teeth is half the periodic time of the fork. In this case, if the synchronism is exact, any point of the slit is seen as two points in the two positions it occupies when illumination takes place, and the whole slit appears as in fig. 12. If the synchronism is not exact, the chain figure appears to move in one direction or the other according as the interval between the passage of successive teeth is greater or less than half the periodic time of the fork.

The advantage of using the second figure rather than the first, is that it is easier by reference to the cross wire at the centre of the slit to tell when the figure is stationary and the synchronism exact.

The box surrounding the fork is blackened; and two holes, one in front, and one behind, give the observer a line of vision.

The observations are made with the naked eye.

For the chain figure we have, when synchronism is exact,

$$n = \frac{2\text{P}}{\text{N}}$$

where n = the number of revolutions of the disc per second,
N = number of teeth in the row used,
P = vibration frequency of the tuning fork.

When a first adjustment has been made and the figure is nearly stationary, the direction of its motion indicates whether the rate of rotation is too quick or too slow, and the requisite change of resistance to obtain synchronism is made by the observer.

The fork was not maintained in vibration electrically, but bowed. It would be more convenient in observing, to use an electrically maintained fork; but preliminary measurement of the pitch of a fork and of a reed, maintained electrically in vibration, tended to throw doubt on them as time-keepers (*vide* 'Phil. Mag.,' April, 1889, "On the Use of LISSAJOUS' Figures to Determine a Rate of Rotation, and of a Morse Receiver to Measure the Periodic Time of a Reed or Tuning Fork.")

By the above method, the rate of rotation of the disc is controlled and referred to the fork during the observations.

It remains to indicate how the pitch of the fork was determined. This was effected by an inverse process. If, while synchronism is maintained between the rotating apparatus and the fork by keeping the figure stationary, the number of revolutions in a given time is determined, the pitch of the fork may be calculated from the formula

$$P = \frac{nN}{2}.$$

In the present experiments the number of revolutions of the cylinder in a given time was determined by a telegraphic method.

An eccentric ring, fixed on the axle at J (Plate 1, fig. 2), communicates a vibratory motion in a vertical plane to a small lever horizontal in its mean position. To the end of the lever is attached a piece of ebonite, through which passes an arched piece of platinum wire, under the ends of which are two mercury cups (η η); and the mercury cups are so adjusted that, for about half the revolution period, the ends of the platinum wire dip in the mercury, and for the other half are lifted above it. The circuit of one pen of a BAIN's Electro-Chemical Telegraph Receiver is completed when the ends of the platinum wire are in the mercury, and broken when they are out. A series of dashes and spaces is therefore made on the telegraph tape when the disc is rotated, a dash and a space corresponding to one revolution. Side by side with the record of the revolutions, a second pen marks the record of the laboratory standard clock. In the clock, the teeth of the escapement wheel lift a lever once a second, thereby breaking contact in the circuit of the second pen of the receiver. Lest inequality in the cutting of the teeth of the escapement wheel should introduce an error, the record of an integral multiple of a minute has been taken in the determinations. To determine the vibration frequency of the fork, it is only necessary to maintain the figure chosen stationary for three or four minutes while the clock and rotating apparatus make their records side by side upon the tape. Counting the tape, we obtain the rate of rotation of the cylinder, and hence, by the formula, the vibration frequency of the fork.

The results of two determinations of the vibration frequency of the fork by this method are given in § 27.

§ 11. The measuring machine was made by Sir J. WHITWORTH and Co., and is similar to the ten-thousandth machines ordinarily made by them, except that its bed is of unusual length. It will receive and measure lengths up to 40 inches. To the loose headstock Y (Plate 3, figs. 17, 18) is attached an arm Ar, which projects at right angles to the length of the bed, and in it the movable electrode O is fixed. A screw to move the loose headstock runs along the centre of the graduated bed, and carries at the end, remote from the fixed headstock, a wheel, H (Plate 2, fig. 5), divided on its circumference into 500 parts. The pitch of the screw is ·5 inch, and therefore one division on the wheel, H, corresponds to ·001 inch. The readings for the position of the movable electrode were, during the experiments, taken on this screw and wheel. The screw is, however, not of guaranteed accuracy, and therefore the distance between the equilibrium positions has been subsequently measured by reference to the standard bars. In Table VI. the corrected values of l are obtained in this way, the uncorrected values being the differences of the readings on the long screw.

§ 12. The wires of the battery circuit are indicated in Plate 2, fig. 5. The current enters and leaves the mercury in the trough by iron plates, $p_1 p_2$, placed vertically at the ends of it. The plates are 5 inches long, 1·75 inch broad, and ·25 inch thick. Some trouble was found in keeping the plates when left in contact with the mercury free of rust; they required frequent cleaning.

The battery E consisted of 12 secondary cells coupled in series; and the resistance of the circuit is such that the current used is something less than ·5 ampère.

The wires from the battery are led to the key K_1, and thence to the commutator K_2. The latter is of the form described by the author at the British Association meeting at Aberdeen (*vide* 'Electrician,' vol. 15, p. 370). A wire from the commutator goes through the standard coil to the iron plate, p_1, at one end of the trough; and from the plate, p_2, at the other end, a wire passes through the coil U_2 to the commutator. The object of the coil U_2 will be subsequently explained.

The wires of the disc and galvanometer circuit are also shown in Plate 2, fig. 5. The brushes are connected by flexible pieces of copper tape to the ends of the stiff cable of the circuit. The cable is seven strands, No. 16 B.W.G. copper wire, thickly insulated with vulcanised rubber. The cable from the central brush passes to the movable electrode O in the trough; from the fixed electrode P it passes through the coil U_1 (the object of which is explained hereafter, and which consists of only two or three turns) to the galvanometer key, K_3, and thence to the copper tape attached to the outside brush.

Great care was taken in insulating the battery and galvanometer circuits.

The central brush was placed to earth through a gas pipe.

The electrodes are of platinum wire ·04 inch in diameter, and descend vertically into the mercury, the lower ends being bent so as to be horizontal for about ·25 inch. Before they were placed in position the end of each was pointed, and the whole wire was thickly covered with paraffin wax. The wax was then shaved away at the end until the platinum point only just emerged so as to secure as nearly as possible a definite point of contact between the electrode and the mercury. The height of the electrode points above the bottom of the trough was about ·5 inch.

§ 13. *Spherometer.*—The spherometer is shown drawn quarter full size in Plate 3, figs. 9 and 10. It consists of a steel screw V, of pitch $\frac{1}{14}$-inch, supported on a stiff iron stand, provided with three levelling screws. The large head h h, on the top is divided into 360 divisions, and a scale s s, is provided to mark complete turns.

The end of the screw is turned down to a very fine point and highly polished.

A mercury cup d, is provided, by means of which a wire coming from a battery through a galvanometer could be placed in electrical contact with the screw, the other wire from the battery being soldered to the iron plate p_1 (Plate 2, fig. 5), and thereby placed in electrical communication with the mercury in the trough.

The position of the mercury surface in the trough is determined by moving the spherometer screw downwards until the passage of a current through the galvanometer indicates that the circuit is complete.

It is of the utmost importance that the point of the screw should be kept clean, otherwise successive observations may differ from one another as much as ·001 inch; but, with clean mercury, if the point is carefully wiped with filter paper before making the observations, half-a-dozen observations may be without difficulty obtained, none of which differ from the mean by more than ·00005 inch. One division on the head is equal to $\frac{1}{5040}$ inch, and the head is large enough to allow of an accurate estimation of tenths of a division.

To preserve the point, sparking at the mercury surface should be reduced as much as possible. With this view, a large resistance is inserted in the circuit, and, between successive observations, contact is broken at the galvanometer before the screw is raised.

Details of the calibration of the spherometer screw are given in § 29. The mercury used has been several times treated with nitric acid and distilled.

§ 14. *Thermometers.*—The thermometers are three in number, one at each end of the trough, and one between the prongs of the fork. They were made by NEGRETTI and ZAMBRA, and are graduated to tenths of a degree. The two trough thermometers were sent to Kew for correction, and it is the corrected temperatures that are inserted in Tables I.–V.*

§ 15. It has already been pointed out that the trough must suffer distortion if the temperature varies, the breadth towards the central part becoming less as the temperature increases. After the experiments were concluded, a measurement was taken at 19·2° C., and the breadth of the trough was found to be 1·51362 inch, the breadth at 15·5° C. being 1·51681 inch. This gives a decrease of breadth of ·000862 per degree or

$$b_t = b_{15\cdot5}\,(1 - \cdot000568\,\overline{t - 15\cdot5}).$$

If, then, the temperature of the paraffin wax could be determined with accuracy, a correction might be made in each set of observations for that temperature. But the wax is a poor conductor of heat, and it takes a long time to assume the temperature of the medium in which it is placed, and different portions of it are likely to be at different temperatures, unless the temperature of the surrounding space has remained for a very long time constant. An attempt was, therefore, made to keep a temperature as constant as possible during the course of the observations.

The trough and measuring machine were enclosed in a wooden box X, Plate 2, fig. 5, covered with several layers of felt paper, and felt curtains were slung round it from a beam above. The thermometers in the trough could be read through windows in the box by lifting the curtain; and everything was so arranged that a set of observations could be taken without opening the box further than to remove for a short time one of the glass windows. The windows were eight inches high by two inches broad.

* *Vide* note, p. 10.

The room outside was kept as nearly as could be at 15°·5 C., which was taken as standard temperature throughout the observations. The extreme variation of temperature in the box did not amount to more than half a degree during the month in which the observations were taken.

The motor, bearings, coil, trough, and measuring machine all rest on slate slabs placed on brick pillars built on the four feet of concrete forming the floor of the laboratory. It is of great importance that the trough supports should be such as to free it from all vibration; otherwise waves on the mercury render the spherometer readings uncertain.

§ 16. *The Galvanometer.*—The galvanometer used was a Thomson reflecting galvanometer, made by ELLIOTT, of resistance ·968 ohm. The suspension, however, was altered. The vertical brass-piece ordinarily bearing the compensating magnet was replaced by a glass tube surmounted by brass fittings giving the usual suspension adjustments. The suspending fibre was a quartz thread about thirteen inches long, one of a number kindly sent to the author by Mr. BOYS. As suspending fibres for galvanometer needles, it is difficult to praise these fibres too highly. They will give a constant zero in a field many times weaker than the weakest in which it would be possible to use silk fibres. With the galvanometer in question the period of oscillation with a quartz fibre could be made as much as twenty-one or twenty-two seconds, while still maintaining a zero as constant as silk gave with a period of about four seconds.

In the course of the experiments the oscillation period was about eight seconds.

The distance of the scale S (Plate 2, fig. 5) from the galvanometer was forty inches. The divisions of the scale are forty to the inch; and the readings were taken through a large magnifying glass.

Adjustment of the Instruments and Method of Observation.

§ 17. The first operation was to place the frame supporting the axle of the rotating disc in such a position that the axis of rotation should be approximately at right angles to the magnetic meridian, and the plane of the disc, therefore, in the plane of the meridian. The object was to get rid, as far as possible, of any induction current in the disc due to a magnetic field, other than that of the current through the standard coil. In the event it proved that this current was so small as to need no special compensating apparatus during the observations. When no current was passing through the standard coil, the deflection obtained by rotating the disc did not amount to more than could easily be counteracted by a slight movement of the compensating magnet placed on the table at some two feet distance from the galvanometer.

Attention was next directed to the four bearings β, β, β, β, Plate 2, fig. 5, so as to ensure smooth and steady rotation. They were carefully scraped to correct any slight inequalities introduced by bolting them down to the brick pillars, and a sight feed lubricator giving about three drops per minute was fitted to each.

The shaft of the electromotor was coupled to the axle by a flexible coupling g, Plate 2, fig. 5 ; and the disc was then ground true in place as mentioned in § 7.

§ 18. The adjustment of the standard coil so that it should be co-axial with the disc, and that its mean plane should coincide with the mean plane of the disc, was made with the help of the circular line inside the coil cylinder, the cutting of which has been already described in § 3. To effect this an arm, $f\,f$, Plate 3, figs. 13 and 14, of adjustable length, had been prepared to fit the disc. The face at e was scraped plane so that when the arm was firmly clamped in the position shown in figs. 13 and 14, the plane of the face as nearly as possible coincided with the mean plane of the disc. Great care was taken to effect this by continually reversing the arm during the scraping and testing with the help of a straight edge and feeling piece.

It will be subsequently shown that the effect of any possible small error in the accuracy with which this was accomplished is negligible.

The adjustment of the standard coil consists in placing the coil so that, when the arm on the disc is adjusted to proper length, the edge of its face e travels, when the disc is turned, over the circle cut on the inside surface of the coil cylinder in the mean plane of the coil (*vide* § 3). The coil is first moved into position as nearly as may be by eye, and the final adjustments are made by turning the levelling screws l, l, l, Plate 1, figs. 1 and 2, and by taps on the coil frame, which move either the levelling screws in the cups over the bases of the cups, or the cups as a whole over the slate slab.

It has already been observed that the coil cylinder is slightly oval, and hence the arm could only be made to touch it at two points at the ends of the shortest diameter. It will be seen from § 24 that the difference between the longest and shortest radius is about ·008 inch.

§ 19. To adjust the mercury trough its base must be levelled and its lateral faces made vertical and parallel to the length of the bed of the measuring machine. The trough was not supported so as to give the requisite number of independent movements to effect these adjustments. It was therefore brought into position by successive small movements, the levelling of the base in the direction of its length being tested by a delicate spirit level, and the verticality of the side faces by the same level with the help of an accurate steel square, the stock of which was pressed up against the face. The parallelism of the faces to the run of the measuring machine was tried by fixing in the electrode arm, Ar, Plate 3, fig. 18, a turned cylinder and inserting a wooden wedge between the cylinder and the face. If the reading on the wedge is the same when the cylinder is moved from one end of the trough to the other, the required parallelism is obtained. Further by making the reading when the wedge was inserted between the cylinder and the one face the same as when it was inserted between the cylinder and the other, the axis of the cylindrical hole was brought into the median plane of the trough. The electrode subsequently inserted was approximately axial to the hole, and therefore nearly in this median plane.

The spherometer was adjusted so that the divided head plate was level both when the instrument was used and when it was calibrated (*vide* § 29).

§ 20. The apparatus being in place and the connecting wires added, it was found that while the galvanometer circuit was broken, a slight deflection of the galvanometer needle accompanied a change in the direction of the current through the battery circuit. A compensating coil U_2 (Plate 2, fig. 5), consisting of nearly three turns of the wire of the battery circuit, satisfactorily counterbalanced this disturbance of the magnetic field at the needle.

§ 21. Tests for insulation were next made with an E.M.F. of 24 volts.

The insulation resistance between the wire of the coil and the coil cylinder was found to be more than 50 megohms, that between the disc and axle more than 100 megohms, and that of the completed circuits (the cups under the levelling screws of the coil being on thin paraffined ebonite plates) more than 70 megohms.

§ 22. The observations consist fundamentally in the determination of equilibrium positions of the movable electrode, together with the rates of rotation to which they correspond (*vide* § 4). In determining an equilibrium position, a definite rate of rotation was first obtained by the observer at the tuning-fork and maintained by him, as accurately as possible, throughout the determination, an assurance of stationariness being received from him by the observer at the galvanometer when the readings were being taken.

It was found best, following Lord RAYLEIGH ('Phil. Trans.,' 1883, "Experiments by the Method of LORENZ to determine the Value of the B.A. Unit of Resistance"), to take two sets of galvanometer readings for each equilibrium position, one set giving the change of galvanometer reading corresponding to reversal of the battery current for a position slightly to one side of the equilibrium position; the other, the change of galvanometer readings corresponding to reversal for a second position slightly on the other side of the equilibrium position.

The galvanometer readings corresponding to the two positions of the commutator being called E and W, passage through an equilibrium position is indicated by a change in the algebraic sign of $W - E$. When $W - E$ has been found for two positions slightly differing from one another, and including between them the equilibrium position, the latter may be deduced by simple interpolation.

Since the readings E and W are not quite fixed owing to small changes in the speed of the disc and slight variations at the brush contacts, it is necessary to take a number of reversals and to find $W - E$ as the mean of the values obtained by combining the first value of E with the first of W, the second of E with the first of W, the second of E with the second of W, &c. The more quickly the reversals can be made to succeed one another the better will be the results, for with rapid reversals, variations in the position of the needle corresponding to no current through the standard coil are less likely to supervene. It is, therefore, best not to wait for the needle to come, even approximately, to rest after the disturbance due to the induction current on reversal, but to take the readings for the extreme positions in an oscillation, and, having previously found the co-efficient of damping, to calculate the position of rest from these two readings.

The coil U_1 (Plate 2, fig. 5) is used to adjust the induction so that the swing of the needle at reversal may be of suitable magnitude for this purpose.

It is advantageous to use the same portion of the trough for the observations at both depths of mercury; any slight irregularities in the base of the trough are then without effect. The rates of rotation and the depths requisite to effect this were, therefore, determined by preliminary experiments. It was found that if the depths are approximately ·75 and 1·25 inch that by the use of the rows of 8 and 13 teeth on the drum at the smaller depth, and of 13 and 21 teeth at the greater depth, the required result was secured.

The thermometers in the trough were read immediately before and immediately after each pair of sets of galvanometer readings determining an equilibrium position, and the thermometer between the prongs of the tuning-fork before and after each set.

The measurements on the spherometer were made directly before and after the addition of the mercury required to raise the level in the trough from the first to the second height. The temperatures of the trough thermometers when each measurement was taken were observed and recorded. Each spherometer measurement recorded in Tables I.–V.* is the mean of from four to six observations, the extreme observations differing from each other by not more than ·00003 inch.

Details of Measurement and Calculation.

§ 23. The diameter of the disc was measured in the Whitworth measuring machine. Measurements were made in four directions, approximately equally inclined to each other, with the following results:—

Diameter.	Measurement.
°	inches.
0–180	12·98954
45–225	12·98979
90–270	12·98949
135–315	12·98951
	Mean 12·98958

The temperature at which the measurements were made was 12° C.

* *Vide* note, p. 10.

Hence the radius of the disc at 12° C. = 6·49479 inches, and correcting for temperature we have

the radius at 15°·5 C. = 6·49493 inches.

§ 24. The diameter of the coil was determined in similar fashion, but on account of the silk covering of the wire, the measurement could not be carried to the same degree of accuracy as in the case of the disc. For this reason, and also because of the ellipticity of the coil cylinder the measurements were much more numerous, and were made along 18 diameters. The cylinder while still in the lathe had been divided along its edge into intervals of 10°.

Diameter.	Measurement.
	inches.
0°–180	21·0898
10–190	21·0929
20–200	21·0951
30–210	21·0933
40–220	21·0960
50–230	21·0998
60–240	21·1017
70–250	21·1026
80–260	21·1044
90–270	21·1038
100–280	21·1056
110–290	21·1041
120–300	21·1014
130–310	21·0979
140–320	21·0945
150–330	21·0924
160–340	21·0900
170–350	21·0910
	Mean 21·09757
	t = 17° C.

The mean of a large number of measurements of the diameter of the wire was ·02191 inch.

The measurements were taken at points equidistant, or nearly so, along a length of about 10 yards of wire.

Subtracting the diameter of the wire from the measured outside diameter of the coil we obtain 21·07566 as the mean diameter from the axis of the wire on one side to the axis of the wire on the other: and hence,

the mean radius at 17° C. = 10·53783, and correcting for temperature,
the mean radius at 15°·5 C. = 10·53774.

Calculation of the Induction Coefficient.

§ 25. It has been shown by the author* that the coefficient of mutual induction of a circle and co-axial helix, the circle being in the mean plane of the helix, and the helix consisting of a whole number of turns (n), is given by the formula :—

$$\mathrm{M} = -8n\varpi\frac{\mathrm{A}a}{\mathrm{A}+a}\left\{\mathrm{P}_0 - \tfrac{1}{2}\cdot\tfrac{1}{3}\left(\frac{x}{\mathrm{A}+a}\right)^2\mathrm{P}_1 + \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{1}{5}\left(\frac{x}{\mathrm{A}+a}\right)^4\mathrm{P}_2 - \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{5}{6}\cdot\tfrac{1}{7}\left(\frac{x}{\mathrm{A}+a}\right)^6\mathrm{P}_3 + \&c.\right\}$$

$$= -8n\varpi\frac{\mathrm{A}a}{\mathrm{A}+a}\Sigma(-1)^m\frac{1.3.5\ldots\overline{2m-1}}{2.4.6\ldots 2m}\,\frac{1}{2m+1}\left(\frac{x}{\mathrm{A}+a}\right)^{2m}\mathrm{P}_m \quad . \quad . \quad . \quad . \quad (\mathrm{I.})$$

where A = the radius of a circular section of the cylinder determined by the helix,

a = the radius of the circle,
$2x$ = the axial length of the helix,

and

$$\mathrm{P}_m = \int_0^{\varpi/2}\frac{\cos 2\theta\, d\theta}{(1-c^2\sin^2\theta)^{(2m+1)/2}},$$

c being equal to $\dfrac{2\sqrt{(\mathrm{A}a)}}{\mathrm{A}+a}$.

As the formula has not previously been used for the calculation of the induction coefficient, it may be of utility to indicate the method of calculation that has been found convenient. For our present purpose it will be sufficient if six terms of the series are found.

Let

$$\mathrm{Q}_m = \int_0^{\varpi/2}\frac{d\theta}{(1-c^2\sin^2\theta)^{(2m+1)/2}};$$

then

$$\mathrm{P}_m = \left(1-\frac{2}{c^2}\right)\mathrm{Q}_m + \frac{2}{c^2}\mathrm{Q}_{m-1} \quad . \quad . \quad . \quad . \quad (\mathrm{II.})$$

and

$$\mathrm{Q}_m = \mathrm{Q}_{m-1} + \frac{c}{2m-1}\,\frac{d}{dc}\mathrm{Q}_{m-1} \quad . \quad . \quad . \quad . \quad (\mathrm{III.})$$

Observing that

$$\mathrm{Q}_0 = \mathrm{F}$$

and

$$\mathrm{Q}_{-1} = \mathrm{E},$$

* "On the Calculation of the Coefficient of Mutual Induction of a Circle and a Coaxal Helix," communicated to the Physical Society in November, 1888 ('Phil. Mag.', Jan., 1889).

where F and E are the complete elliptic integrals to modulus c, and using the relation

$$\frac{d\mathrm{F}}{dc} = \frac{\mathrm{E}}{c(1-c^2)} - \frac{\mathrm{F}}{c}$$

we have

$$\mathrm{Q}_1 = \frac{\mathrm{E}}{1-c^2}\,.$$

The calculation may now be conducted systematically as follows :—

(i.) Calculate

$$\mathrm{Q}_1, \quad \frac{d\mathrm{Q}_1}{dc}, \quad \frac{d^2\mathrm{Q}_1}{dc^2}, \quad \frac{d^3\mathrm{Q}_1}{dc^3}, \quad \&c.$$

or denoting differentiation with regard to c by a dot

$$\mathrm{Q}_1, \ \dot{\mathrm{Q}}_1, \ \ddot{\mathrm{Q}}_1, \ \dddot{\mathrm{Q}}_1, \ \&c.$$

These may be successively calculated from the equation

$$(1-c^2)\,\mathrm{Q}_1 = \mathrm{E},$$

and its derivatives

$$c\,(1-c^2)\,\dot{\mathrm{Q}}_1 - 2c^2\mathrm{Q}_1 = \mathrm{E} - \mathrm{F},$$

$$c\,(1-c^2)\,\ddot{\mathrm{Q}}_1 + \dot{\mathrm{Q}}_1\,(1-5c^2) - 3c\mathrm{Q}_1 = 0,$$

$$c\,(1-c^2)\,\dddot{\mathrm{Q}}_1 + 2(1-4c^2)\,\ddot{\mathrm{Q}}_1 - 13c\dot{\mathrm{Q}}_1 - 3\mathrm{Q}_1 = 0,$$

$$c\,(1-c^2)\,\ddddot{\mathrm{Q}}_1 + (3-11c^2)\,\dddot{\mathrm{Q}}_1 - 29c\ddot{\mathrm{Q}}_1 - 16\dot{\mathrm{Q}}_1 = 0.$$

(ii.) Calculate

$$\mathrm{Q}_2, \ \dot{\mathrm{Q}}_2, \ \ddot{\mathrm{Q}}_2, \ \dddot{\mathrm{Q}}_2$$

from the equation

$$\mathrm{Q}_2 = \mathrm{Q}_1 + \frac{c}{3}\,\dot{\mathrm{Q}}_1,$$

and its derivatives

$$\dot{\mathrm{Q}}_2 = \tfrac{4}{3}\,\dot{\mathrm{Q}}_1 + \frac{c}{3}\,\ddot{\mathrm{Q}}_1,$$

$$\ddot{\mathrm{Q}}_2 = \tfrac{5}{3}\,\ddot{\mathrm{Q}}_1 + \frac{c}{3}\,\dddot{\mathrm{Q}}_1,$$

$$\dddot{\mathrm{Q}}_2 = \tfrac{6}{3}\,\dddot{\mathrm{Q}}_1 + \frac{c}{3}\,\ddddot{\mathrm{Q}}_1.$$

(iii.) Calculate

$$\mathrm{Q}_3, \ \dot{\mathrm{Q}}_3, \ \ddot{\mathrm{Q}}_3$$

from the equation

$$\mathrm{Q}_3 = \mathrm{Q}_2 + \frac{c}{5}\,\dot{\mathrm{Q}}_2,$$

and its derivatives

$$\dot{Q}_3 = \tfrac{6}{5}\,\dot{Q}_2 + \frac{c}{5}\,\ddot{Q}_2,$$

$$\ddot{Q}_3 = \tfrac{7}{5}\,\ddot{Q}_2 + \frac{c}{5}\,\dddot{Q}_2.$$

(iv.) Calculate Q_4, $\dot{Q}_4$ from the equation

$$Q_4 = Q_3 + \frac{c}{7}\,\dot{Q}_3$$

and its derivative

$$\dot{Q}_4 = \frac{8}{7}\,\dot{Q}_3 + \frac{c}{7}\,\ddot{Q}_3\,.$$

(v.) Calculate Q_5 from the equation

$$Q_5 = Q_4 + \frac{c}{9}\,\dot{Q}_4\,.$$

(vi.) Calculate P_0, P_1, P_2, P_3, P_4, P_5 from the values of E, F, Q_1, Q_2, Q_3, Q_4, Q_5, by successive applications of Equation II.

For the coil and the circumference of the disc we have :—

$$A = 10{\cdot}53774$$

$$a = 6{\cdot}49493$$

$$\log c = \bar{1}{\cdot}9874084$$

$$E = 1{\cdot}0665493$$

$$F = 2{\cdot}8509606$$

$Q_1 =$	$1{\cdot}8931552 \times 10$
$\dot{Q}_1 =$	$6{\cdot}2026914 \times 10^2$
$\ddot{Q}_1 =$	$4{\cdot}3150869 \times 10^4$
$\dddot{Q}_1 =$	$4{\cdot}5196380 \times 10^6$
$\ddddot{Q}_1 =$	$6{\cdot}3189387 \times 10^8$
$Q_2 =$	$2{\cdot}1977948 \times 10^2$
$\dot{Q}_2 =$	$1{\cdot}4799606 \times 10^4$
$\ddot{Q}_2 =$	$1{\cdot}5354118 \times 10^6$
$\dddot{Q}_2 =$	$2{\cdot}1365132 \times 10^8$
$Q_3 =$	$3{\cdot}0951221 \times 10^3$
$\dot{Q}_3 =$	$3{\cdot}1606635 \times 10^5$
$\ddot{Q}_3 =$	$4{\cdot}3658744 \times 10^7$
$Q_4 =$	$4{\cdot}6957152 \times 10^4$
$\dot{Q}_4 =$	$6{\cdot}4199487 \times 10^6$
$Q_5 =$	$7{\cdot}3990015 \times 10^5$

$$P_0 = - \cdot 93092203$$
$$P_1 = - 1 \cdot 5149680 \times 10$$
$$P_2 = - 2 \cdot 0589783 \times 10^2$$
$$P_3 = - 2 \cdot 9988831 \times 10^3$$
$$P_4 = - 4 \cdot 6004090 \times 10^4$$
$$P_5 = - 7 \cdot 2872416 \times 10^5$$

$$\tfrac{1}{2} \cdot \tfrac{1}{3} \left(\frac{x}{A + a}\right)^2 P_1 = - \cdot 04654257$$

$$\tfrac{1}{2} \cdot \tfrac{3}{4} \cdot \tfrac{1}{5} \left(\frac{x}{A + a}\right)^4 P_2 = - \cdot 00524698$$

$$\tfrac{1}{2} \cdot \tfrac{3}{4} \cdot \tfrac{5}{6} \cdot \tfrac{1}{7} \left(\frac{x}{A + a}\right)^6 P_3 = - \cdot 00083851$$

$$\tfrac{1}{2} \cdot \tfrac{3}{4} \cdot \tfrac{5}{6} \cdot \tfrac{7}{8} \cdot \tfrac{1}{9} \left(\frac{x}{A + a}\right)^8 P_4 = - \cdot 00016136$$

$$\tfrac{1}{2} \cdot \tfrac{3}{4} \cdot \tfrac{5}{6} \cdot \tfrac{7}{8} \cdot \tfrac{9}{10} \cdot \tfrac{1}{11} \left(\frac{x}{A + a}\right)^{10} P_5 = - \cdot 00003469$$

$$\therefore \Sigma (-1)^m \frac{1 \, . \, 3 \, . \, 5 \ldots \overline{2m - 1}}{2 \, . \, 4 \, . \, 6 \ldots 2m} \frac{1}{2m + 1} \left(\frac{x}{A + a}\right)^{2m} P_m = - \cdot 88891460$$

and $M = 89 \cdot 7717 \, n$.

Two small corrections are needed in order to obtain from the above value of M the actual coefficient of mutual induction used in the experiments, viz. the correction for the central brush, and the correction for the iron in the field.

The radius of the central brush $= \cdot 05847$ inch : and the coefficient of mutual induction of its circumference and the standard coil $= \cdot 00626 \, n$. Subtracting this from the coefficient of mutual induction of the circumference of the disc and the coil we have $M = 89 \cdot 7654 \, n$.

The number of turns n on the coil is 185,

$$\therefore M = 16606 \cdot 60.$$

The iron in the field consists of the steel axle $\xi \, \xi$ (Plate 2, fig. 5), the steel fly-wheel, and the iron of the motor. The fly-wheel and motor iron will, on account of their distance, hardly affect the result to a greater extent than the probable error of the mean of the five sets of observations (Tables I.–V.). But the effect of the steel in the part of the axle $\xi \, \xi$ is appreciable. It was estimated and allowed for as follows. A steel bar of the same dimensions was prepared which could be placed in a position on the other side of the coil and disc symmetrical with regard to their mean plane to that occupied by the axle $\xi\,\xi$. A series of observations were then made of the equilibrium positions of the movable electrode with the steel bar in and out of place, the rate of rotation and the level of the mercury in the trough being constant, and proper corrections being introduced for temperature in the manner indicated later in § 31.

Let

M = the coefficient of mutual induction with the steel bar away.

M + ΔM = the coefficient of mutual induction with the steel bar in place.

L = the initial distance between the two electrodes.

ΔL = the distance moved through by the movable electrode to its new equilibrium position when the steel bar is introduced into the field.

Then

$$\frac{\Delta M}{M} = \frac{\Delta L}{L}.$$

In the observations made

$$\begin{aligned} L &= 26{\cdot}8615, \\ \Delta L &= {\cdot}01157, \\ \therefore\ \Delta M &= {\cdot}0004307\ M, \\ &= 7{\cdot}153. \end{aligned}$$

This may be taken as a sufficiently close approximation to the correction that must be added to the calculated coefficient of mutual induction in consequence of the steel axle $\xi\,\xi$. Making this correction, we have as the final value of the coefficient used in the experiments

$$M = 16613{\cdot}75.$$

A calculation was made to find the effect of the finite thickness of the wire of the coil on the coefficient of induction. In the formula it is assumed that the current is concentrated in the axis of the wire. Calculation showed that the mean of the induction coefficients, obtained on the suppositions that the current is concentrated on the outside and inside of the wire respectively, does not differ from the induction coefficient calculated on the hypothesis that the current is concentrated in the axis of the wire by more than, approximately, 1 part in 300,000. In the experiments we have the current uniformly distributed over the section of the wire, and the error introduced by supposing the current concentrated in the axis is less than the error calculated in the extreme case taken, and for present purposes quite negligible.

§ 26. It seems proper here to consider the effect on the coefficient of mutual induction of a possible non-coincidence of the mean planes of the coil and disc.

In making the adjustment (§ 18), the edge of the face *e* of the arm *f f*, Plate 3, figs. 13 and 14, is made to travel over the circular line cut as described in § 3 on the inside of the coil cylinder. If the face e does not coincide with the mean plane of the disc, and if the plane of the line on the coil cylinder does not coincide with the mean plane of the coil, the adjustment does not bring the mean planes into coincidence, but into positions such that the distance between the mean planes is equal to the algebraical sum of the distance between the plane of the circle described by the edge of e and the mean plane of the disc, and the distance between the plane of the circular line in the coil cylinder and the mean plane of the coil. Having regard to the precautions taken in drawing the line and preparing the arm, the sum of these errors cannot be as much as ·01 inch.

If the distance between the mean planes is assumed to be half the pitch of the helix, the coefficient of mutual induction may be calculated as follows :—we may consider the coefficient in this case to be made up of two parts, viz., the coefficient of mutual induction of the circumference of the disc and of the portion of the coil of axial length $= x + x/n$ on one side of it, and the coefficient of mutual induction of the circumference of the disc and the portion of the coil of axial length $= x - x/n$ on the other side of it. Calling the first of these M_1, and the second M_2, we have

$$M_1 = -4n\varpi \frac{Aa}{A+a} \Sigma(-1)^m \frac{1.3.5\ldots\overline{2m-1}}{2.4.6\ldots 2m} \frac{1}{2m+1} P_m \left(\frac{x}{A+a}\right)^{2m} \left(1+\frac{1}{n}\right)^{2m+1},$$

$$M_2 = -4n\varpi \frac{Aa}{A+a} \Sigma(-1)^m \frac{1.3.5\ldots\overline{2m-1}}{2.4.6\ldots 2m} \frac{1}{2m+1} P_m \left(\frac{x}{A+a}\right)^{2m} \left(1-\frac{1}{n}\right)^{2m+1},$$

$\therefore M_1 + M_2$

$$= -4n\varpi \frac{Aa}{A+a} \Sigma(-1)^m \frac{1.3.5\ldots\overline{2m-1}}{2.4.6\ldots 2m} \frac{1}{2m+1} P_m \left(\frac{x}{A+a}\right)^{2m} \left\{\left(1+\frac{1}{n}\right)^{2m+1} + \left(1-\frac{1}{n}\right)^{2m+1}\right\}$$

$$= -8n\varpi \frac{Aa}{A+a}\left[P_0 - \tfrac{1}{2}\cdot\tfrac{1}{3}\left(\frac{x}{A+a}\right)^2 P_1\left(1+\frac{3}{n^2}\right) + \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{1}{5}\left(\frac{x}{A+a}\right)^4 P_2\left(1+\frac{10}{n^2}+\frac{5}{n^4}\right)\right.$$
$$\left. - \tfrac{1}{2}\cdot\tfrac{3}{4}\cdot\tfrac{5}{6}\cdot\tfrac{1}{7}\left(\frac{x}{A+a}\right)^6 P_3\left(1+\frac{21}{n^2}+\frac{70}{n^4}+\frac{7}{n^6}\right) + \&c.\right]$$

$$= -8n\varpi \frac{Aa}{A+a}(-{\cdot}88891460 + {\cdot}00000408 - {\cdot}00000154 + {\cdot}00000051 - {\cdot}00000017 + {\cdot}00000006)$$

$$= -8n\varpi \frac{Aa}{A+a}(-{\cdot}88891460 + {\cdot}00000294)$$

$$= M(1 - {\cdot}00000331).$$

Or if the distance between the mean planes amounts to one half the pitch of the helix, *i.e.*, to ·0125 inch, the coefficient of mutual induction is diminished only by about one part in 300,000.

Hence there can be no appreciable error due to this cause.

Determination of the Vibration Frequency of the Tuning Fork.

§ 27. The details of two measurements of the vibration frequency of the fork by the method described in § 10 are given in the following tables. To the degree of accuracy required the clock needed no correction.

Temperature of fork.	Seconds.	Tape reading.	Seconds.	Tape reading.	Revolutions in three minutes.
Initial 14·98	0	1·72	180	2917·10	2915·38
	1	17·97	181	2933·30	2915·33
	2	34·09	182	2949·42	2915·33
	3	50·41	183	2965·95	2915·54
	4	66·44	184	2981·82	2915·38
Final 15·01	5	82·78	185	2998·26	2915·48
Mean 14·995	..	..	..	Mean	2915·407

Hence number of revolutions per minute . = 971·802
And „ „ „ „ second . = 16·1967
And P = 64·7868, or, corrected* to 15°·5, P = 64·7832

Temperature of fork.	Seconds.	Tape reading.	Seconds.	Tape reading.	Revolutions in three minutes.
Initial 15·06	0	2·0	180	2917·37	2915·37
	1	17·96	181	2933·33	2915·37
	2	34·17	182	2949·50	2915·33
	3	49·92	183	2965·25	2915·33
	4	66·54	184	2981·91	2915·37
Final 15·08	5	82·47	185	2997·86	2915·39
Mean 15·07	..	..	..	Mean	2915·360

Hence number of revolutions per minute . = 971·787
And „ „ „ „ second . = 16·19645
And P = 64·7858, or corrected* to 15°·5, P = 64·7828

Mean $P_{15^\circ\cdot 5} = 64\cdot7830$.

§ 28. *Calibration of the Trough.*—The form of the callipers used to measure the breadth of the trough is shown in elevation and plan in Plate 3, figs. 15 and 16.

Two side pieces of brass *r*, *r*, with their faces carefully scraped plane and parallel, are suspended by silk threads from the wooden framework of the apparatus. Between

* The correcting coefficient used is that found by MACLEOD and CLARKE ('Phil. Trans.,' 1880), viz., − ·00011.

them are a pair of wedges, w_1, w_2, with their faces scraped plane, and such that, when they are put together, as in the figure, their outside faces are parallel. The wedge w_1 is graduated into twentieths of an inch, and an index mark is placed on the wedge w_2. When the apparatus is in the trough, the centre of the brass pieces r, r, is at the mean height of the mercury surface during the measurements recorded in Tables I.–V. The method of procedure was to use the wedge with the handle attached as a feeling piece, and to push it in until when very slightly lifted it was held in place. Measurements were made every half-inch along the portion of the trough used. At each point four readings were taken which are recorded in the table below. Only in one case do the four readings at any point differ from one another by more than ·2 division (= ·0001 inch approximately). The following table contains the results of the trough measurements :—

Position in trough.	Wedge readings.				Mean.
Inches.					
24·5	33·4	33·4	33·4	33·5	33·42
24·0	34·5	34·6	34·5	34·4	34·50
23·5	35·0	35·0	35·0	35·0	35·00
23·0	34·9	34·9	35·0	34·9	34·92
22·5	35·3	35·3	35·1	35·3	35·25
22·0	34·4	34·4	34·5	34·3	34·40
21·5	34·3	34·2	34·3	34·3	34·28
21·0	33·6	33·7	33·7	33·8	33·70
20·5	33·9	33·9	33·9	34·0	33·92
20·0	33·7	33·7	33·6	33·6	33·65
19·5	33·2	33·2	33·25	33·2	33·21
19·0	33·7	33·7	33·7	33·8	33·72
18·5	34·9	34·9	34·7	34·7	34·80
18·0	35·0	35·0	35·0	35·0	35·00
17·5	35·8	35·7	35·9	35·7	35·78
17·0	35·8	35·9	35·9	36·0	35·90
16·5	35·1	35·2	35·3	35·3	35·22
16·0	33·5	33·6	33·7	33·7	33·62
15·5	32·2	32·0	32·2	32·5	32·22
15·0	34·3	34·4	34·3	34·5	34·38
Mean at 15°·5 C. 34·344					

After the observations in the trough were completed the whole apparatus was transferred to the measuring machine, and the breadth corresponding to given readings of the wedges determined. The breadth corresponding to the mean reading in the trough was then calculated by simple interpolation.

Wedges transferred to measuring machine.	
Wedge readings.	Thickness given by measuring machine.
41·2 36·9	1·5100″ 1·5080
4·3	·002
A decrease in thickness of ·002″ corresponds to a movement of 4·3 divisions on the wedges. ∴ The thickness corresponding to the reading 34·344 of the wedges = 1·51681 inch.	

§ 29. *Calibration of Spherometer Screw.*—For this purpose the wedges and one of the side pieces r were used with a Whitworth Surface Plate. The steel point of the spherometer screw was protected by screwing on an auxiliary brass cover with rounded end.

The surface plate was first levelled. The spherometer was then placed on an independent stand on one side of the plate, so that the screw was over the plate; and the stand was so adjusted for height that when the spherometer head h (fig. 9) was levelled as in § 19, the reading on the head for contact with the wedges when they are placed on the surface plate under the screw point is near the readings for the mercury surface in its lower position given in Tables I.–V.

The reading for the surface of the side piece placed on the wedges will then be near the readings for the mercury surface in its higher position, for in each set of observations the difference of level is nearly ·5 inch, and the breadth of the side piece is about the same.

In order to determine the reading for contact between the auxiliary screw point and either the surface of the side piece on the wedges, or the surface of the wedges alone, the spherometer screw head was moved gradually downwards until when the pieces were made to slide over the surface plate under the screw point a slight mark on the surface resulted from the passage.

A series of sets of readings were in the first place taken for contact between the screw point and the surface of the side piece placed on the wedges, the thickness of the wedges being diminished between successive sets by an amount corresponding to five divisions on the graduated wedge. The part of the series required is incorporated in the following table:—

WEDGES on Surface Plate and Side piece on Wedges.

Wedge readings.	Spherometer readings. I. Turns.	I. Divisions.	II. Turns.	II. Divisions.	III. Turns.	III. Divisions.	Mean. Turns.	Mean. Divisions.	Thickness of wedges by measuring machine.	Thickness of side piece by measuring machine.	Height of spherometer point above surface plate.
									Inch.	Inch.	Inch.
25	29	329·5	29	329·0	29	329·5	29	329·3	·48739	·51242	·99981
20	29	340·5	29	340·0	29	341·0	29	340·5	·48510	·51242	·99752
15	29	352·0	29	352·5	29	352·0	29	352·2	·48294	·51242	·99536

The height of the screw point above the surface plate corresponding to the reading of the spherometer scale is in each case equal to the sum of the thicknesses of the wedges and the side piece, which were separately determined in the measuring machine. These thicknesses are recorded in the sixth and seventh columns in the above table, and their sum, placed in the last column, gives in each case the height of the screw point above the surface plate.

A similar set of measurements was then made on the wedges alone, the side piece being removed. The results are given in the accompanying table :—

WEDGES on Surface Plate.

Wedge readings.	Spherometer readings. I. Turns.	I. Divisions.	II. Turns.	II. Divisions.	III. Turns.	III. Divisions.	Mean. Turns.	Mean. Divisions.	Thickness of wedges by measuring machine.	Height of spherometer point above surface plate.
									Inch.	Inch.
55	36	325·0	36	325·5	36	325·0	36	325·2	·50104	·50104
50	36	337·5	36	338·0	36	338·0	36	337·8	·49883	·49883
45	36	349·0	36	349·0	36	349·0	36	349·0	·49657	·49657
40	37	0	37	0	37	0	37	0	·49423	·49423

The above tables are extracted from a more extended set of measurements. They contain the measurements required in interpreting the readings of the spherometer contained in Tables I.–V.

The following tables were then constructed by simple interpolation from the above results, each spherometer reading having the corresponding height of the screw point above the surface plate attached to it :—

Spherometer readings.		Height (calculated).	Spherometer readings.		Height (calculated).
Turns.	Divisions.	″	Turns.	Divisions.	″
29	330	·999667	36	331	·500023
29	331	·999462	36	332	·499847
29	332	·999258	36	333	·499672
29	333	·999053	36	334	·499497
29	334	·998849	36	335	·499321
29	335	·998644	36	336	·499146
29	336	·998440	36	337	·498970
29	337	·998235	36	338	·498790
29	338	·998031	36	339	·498588
29	339	·997826	36	340	·498386
29	340	·997622	36	341	·498184
29	341	·997428	36	342	·497982
29	342	·997243	36	343	·497781
29	343	·997059	36	344	·497579
29	344	·996874	36	345	·497377
29	345	·996689	36	346	·497175
29	346	·996505	36	347	·496973
29	347	·996320	36	348	·496772
29	348	·996136	36	349	·496570
29	349	·995951	36	350	·496357
29	350	·995766	36	351	·496145
29	351	·995582	36	352	·495932
29	352	·995397	36	353	·495719
			36	354	·495507
36	325	·501075	36	355	·495294
36	326	·500900	36	356	·495081
36	327	·500724	36	357	·494868
36	328	·500549	36	358	·494656
36	329	·500374	36	359	·494443
36	330	·500198	37	0	·494230

In calculating the difference of level of the mercury surfaces from the readings given in Tables I.–V., it is only necessary to take the difference of the two corresponding heights in the spherometer calibration Table given above.

§ 30. A record of the observations is given in Tables I.–V. Each table contains the observations requisite for a complete determination which involves finding four equilibrium positions. The upper part of each table is devoted to the observations giving the two equilibrium positions with the surface of the mercury in the trough at the lower level, the lower part to those giving the two equilibrium positions with the mercury surface at the higher level, and in the middle is placed the record of the spherometer readings, from which the difference of level may be calculated. Each equilibrium position is obtained by simple interpolation from two values of W — E (*vide* § 22), and the positions c_1, c_2, of the movable electrode corresponding to them. The galvanometer readings, from which these two values of W — E are calculated, together with other necessary data, are placed in each case above the line recording the equilibrium position deduced from them.

Above each set of galvanometer readings, the following corresponding observations are given :—

(i.) The reading for the movable electrode, *c*.

(ii.) t_1, t_2, the temperatures of the thermometers T_1, T_2 (Plate 2, fig. 5), taken before or after the galvanometer readings according as the set is marked α or β.

(iii.) θ, the temperature of the thermometer between the prongs of the tuning fork, taken before and after the galvanometer readings.

(iv.) N, the number of teeth in the row on the drum, Dr (Plate 2, fig. 5), seen through the fork slit.

Each set is distinguished by a Roman numeral indicating the table to which it belongs, an Arabic numeral indicating the equilibrium position (1, 2, 3, or 4) it helps to determine, and the letter α or β according as it is the first or second set belonging to that equilibrium position.

In any set of galvanometer readings, the observed readings are contained in the first four columns, the first two columns under the letter E being the readings for an oscillation when the commutator is in the position E, and the third and fourth columns, under the letter W, being the readings for an oscillation when the commutator is in the position W. For instance, in the first set in Table I (I 1α) the readings called out by the observer at the galvanometer followed one another, thus :— − 42, + 23·5, Reversal; + 26·5, − 13, Reversal; − 28·5, + 15·5, Reversal; + 26, − 14, Reversal; − 28·5, + 15, Reversal; + 27·5, − 16, Reversal, &c.

The fifth column under E contains the positions of rest of the galvanometer needle, calculated from the readings on the same line in the first two columns, and the coefficient of damping. The numbers in the sixth column under W are similarly calculated from those in the third and fourth columns.

The value of the coefficient of damping was 0·6757.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE I.—Set No. 1. (3rd April, 1890.)

Number of experiment.	c_1.	t_1.	t_2.	θ.	N.	Number of experiment.	c_2.	t_1.	t_2.	θ.	N.
I (1) α	15 +160	15·45	15·5	15·0 15·1	8	I (1) β	15 +180	15·45	15·5	15·1 15·2	8

E.		W.		E.	W.	W.–E.	E.		W.		E.	W.	W.–E.
−42	+23·5	+26·5	−13	−2·9	+2·9	+5·8	−38	+26	+20	−21	+ ·2	−4·5	−4·7
−28·5	+15·5	+26	−14	−2·2	+2·1	+5·1 +4·3	−26·5	+17·5	+24	−22·5	− ·2	−3·8	−4·3 −3·6
−28·5	+15	+27·5	−16	−2·5	+1·5	+4·6 +4·0	−26	+19·5	+24·5	−19·5	+1·2	−1·8	−5·0 −3·0
−27	+13·5	+27	−16	−2·8	+1·3	+4·3 +4·1	−26	+19·5	+26	−23	+1·2	−3·2	−3·0 −4·4
−28·5	+15·5	+28	−16·5	−2·2	+1·4	+3·5 +3·6	−24	+15	+29	−27	− ·7	−4·4	−2·5 −3·7
−28·5	+15	+26	−16	−2·5	+ ·9	+3·9 +3·4	−27	+15·5	+24	−25·5	−1·6	−5·5	−2·8 −3·9
−30	+13·5	+27	−19	−4·0	− ·5	+4·9 +3·5	−33	+18·5	+41·5	−37	−2·3	−5·3	−3·2 −3·0
−31·5	+17	+23·5	−13·5	−2·6	+1·4	+2·1 +4·0	−24	+12	+25	−25	−2·5	−4·8	−2·8 −2·3
−31	+19	+33·5	−19	−1·2	+2·2	+2·6 +3·4	−25·5	+15	+24	−23·5	−1·3	−4·3	−3·5 −3·0
−26·5	+16	+30	−13·5	−1·1	+4·0	+3·3 +5·1	−28·5	+16·5	+31·5	−29	−1·6	−4·6	−2·7 −3·0
−34·5	+18	+36	−22	−3·2	+1·4	+7·2 +4·6	−26	+15·5	+26	−28	−1·2	−6·2	−3·4 −5·0
−49	+28·5	+17	−10	−2·7	+ ·9	+4·1 +3·6	−32	+18·5	+28	−27	−1·9	−4·8	−4·3 −2·9
					Mean	+4·13						Mean	−3·48

Calculated reading for equilibrium position 15″·17085.

TABLE I.—Set No. 1. (3rd April, 1890)—continued.

Number of experiment.	c_3.	t_1.	t_2.	θ.	N.	Number of experiment.	c_4.	t_1.	t_2.	θ.	N.
I (2) α.	24 +425	15·46	15·5	15·4 15·5	13	I (2) β.	24 +445	15·46	15·5	15·5 15·5	13

E.		W.		E.	W.	W.–E.
−22	+5	+ 9	−11	−5·9	−2·9	+3·0
−22	+5·5	+15	−12·5	−5·6	−1·4	+2·7 +4·2
−21	+6·5	+16·5	−11	−4·6	− ·1	+3·2 +4·5
−23	+7·5	+17·5	−13·5	−4·8	−1·0	+4·7 +3·8
−21	+6	+19	−13·5	−4·9	− ·4	+3·9 +4·5
−24·5	+7	+15	−14	−5·7	−2·3	+5·3 +3·4
−20·5	+8·5	+16·5	−13	−3·2	−1·1	+ ·9 +2·1
− 0·5	+5	+14·5	−13·5	−5·3	−2·2	+4·2 +3·1
−23	+6·5	+13	−12·5	−5·4	−2·2	+3·2 +3·2
−20·5	+4·5	+14·5	−11·5	−5·6	−1·0	+3·4 +4·6
−20·5	+6	+15	−10·5	−4·7	− ·2	+3·7 +4·5
−18·5	+5·5	+15·5	−11·5	−4·2	− ·6	+4·0 +3·6
−19	+6	+14	−12·5	−4·1	−1·8	+3·5 +2·3
					Mean	+3·58

E.		W.		E.	W.	W.–E.
−18	+10	+11·5	−17	−1·3	−5·5	−4·2
−14	+ 7·5	+11	−15·5	−1·2	−4·8	−4·3 −3·6
−18	+10	+10	−15·5	−1·3	−5·2	−3·5 −3·9
−19	+13	+14	−17·5	+ ·1	−4·8	−5·3 −4·9
−17·5	+ 9	+10·5	−20	−1·7	−7·7	−3·1 −6·0
−17	+ 9	+11	−14	−1·5	−3·9	−6·2 −2·4
−17	+ 9	+ 8·5	−19	−1·5	−7·9	−2·4 −6·4
−17	+ 6·5	+ 8	−18	−3·0	−7·5	−4·9 −4·5
−18	+ 7	+ 8·5	−18	−3·1	−7·3	−4·4 −4·2
−15	+ 4·5	+ 8	−19	−3·4	−8·1	−3·9 −4·7
−14·5	+ 5	+ 9	−16·5	−2·9	−6·2	−5·2 −3·3
−18	+ 8·5	+12	−17·5	−2·2	−5·6	−4·0 −3·4
					Mean	−4·29

Calculated reading for equilibrium position 24″·43410.

TABLE I.—Set No. 1. (3rd April, 1890)—*continued.*

	t_1.	t_2.	Mean temperature.
Spherometer reading for mercury surface corresponding to I. (1) and I. (2) 36 + 327°·3 . . . . at	15·46	15·5	15·48
Spherometer reading for mercury surface corresponding to I. (3) and I. (4) 29 + 341°·2 . . . . at	15·55	15·5	15·525

Number of experiment.	c_5.	t_1.	t_2.	θ.	N.	Number of experiment.	c_6.	t_1.	t_2.	θ.	N.
I (3) *a*.	15·5 +485	15·61	15·6	15·5 15·55	13	I (3) β.	15·5 +505	15·61	15·6	15·55 15·65	13

E.		W.		E.	W.	W.–E.	E.		W.		E.	W.	W.–E.
−17	+14	+17	−6·5	+1·5	+3·0	+1·5	−13	+19	+18·5	− 8	+6·1	+2·7	−3·4
−18	+14	+23·5	−9·5	+1·1	+3·8	+1·9 +2·7	−13	+20	+24·5	− 8·5	+6·7	+4·8	−4·0 −1·9
−16·5	+15	+25	−9·5	+2·3	+4·4	+1·5 +2·1	−12·5	+20	+18·5	− 7	+6·9	+3·3	−2·1 −3·6
−13	+14·5	+24·5	−6	+3·4	+6·3	+1·0 +2·9	−11	+18·5	+24·5	−10·5	+6·6	+3 6	−3·3 −3·0
−15·5	+19	+26	−7·5	+5·1	+6·0	+1·2 + ·9	−14	+20	+21·5	− 9	+6·3	+3·3	−2·7 −3·0
−15·5	+18	+24·5	−6·5	+4·5	+6·0	+1·5 +1·5	−12·5	+19	+23	−11	+6·3	+2·7	−3·0 −3·6
−13·5	+15	+24	−8	+3·5	+4·9	+2·5 +1·4	−13	+19	+22	−11	+6·1	+2·3	−3·4 −3·8
−15	+15	+23	−5	+2·9	+6·3	+2·0 +3·4	−13	+16	+20·5	−10·5	+4·3	+2·0	−2·0 −2·3
−15	+16·5	+23·5	−8	+3·8	+4·7	+2·5 + ·9	−14	+20	+21	− 9	+6·3	+3·1	−4·3 −3·2
−18	+15·5	+23·5	−7·5	+2·0	+5·0	+2·7 +3·0	−12·5	+18·5	+25·5	−11	+6·0	+3·7	−2·9 −2·3
−15	+14·5	+24	−6	+2·6	+6·1	+2.4 +3·5	−12	+19	+19·5	− 7	+6·5	+3·7	−2·8 −2·8
−11·5	+15	+24	−7·5	+4·3	+5·2	+1·8 + ·9	−10·5	+17	+21·5	− 7	+5·9	+4·5	−2·2 −1·4
					Mean	+1·99						Mean	−2·91

Calculated reading for equilibrium position 15″·99312.

TABLE I.—Set No. 1. (3rd April, 1890)—*continued*.

Number of experiment.	c_7.	t_1.	t_2.	θ.	N.
I (4) α.	24·5 +340	15·61	15·6	16·1 16·15	21

E.		W.		E.	W.	W.–E.
−10	− ·5	+ 8·5	− 5	−4·3	+ ·4	+4·7
−14	+3	+11	− 5·5	−3·9	+1·2	+4·3 +5·1
−15·5	+7·5	+14·5	− 8	−1·8	+1·1	+3·0 +2·9
−14·5	+3	+10·5	− 8·5	−4·1	− ·8	+5·2 +3·3
−16·5	+5	+11	−10	−3·7	−1·5	+2·9 +2·2
−16·5	+5·5	+14	− 8·5	−3·4	+ ·6	+1·9 +4·0
−15·5	+2	+ 8·5	− 6	−5·1	− ·2	+5·7 +4·9
−11	+ ·5	+ 9	− 3	−4·1	+1·8	+3·9 +5·9
−15	+2	+10	− 8	−4·9	− ·7	+6·7 +4·2
−15	+4	+13·5	−12·5	−3·7	−2·0	+3·0 +1·7
−17	+3	+ 8·5	− 5·5	−5·1	+ ·1	+3·1 +5·2
−15·5	+3·5	+ 9	− 8	−4·2	−1·1	+4·3 +3·1
−13	+4·5	+15	−11·5	−2·6	− ·8	+1·5 +1·8
					Mean	+3·78

Number of experiment.	c_8.	t_1.	t_2.	θ.	N.
I (4) β.	24·5 +360	15·61	15·6	16·15 16·2	21

E.		W.		E.	W.	W.–E.
− 9·5	+3	+ 6	− 9	−2	−3	−1·0
−11	+6·5	+ 8	− 9	− ·6	−2·1	−2·4 −1·5
− 9	+4	+10	−11	−1·2	−2·5	− ·9 −1·3
−10·5	+3·5	+ 4·5	− 5·5	−2·1	−1·5	− ·4 + ·6
− 8	+4·5	+ 5	− 7·5	− ·5	−2·5	−1·0 −2·0
− 9·5	+8·5	+ 8·5	− 8·5	+1·2	−1·6	−3·7 −2·8
− 8	+5	+ 8	− 8·5	− ·2	−1·8	−1·4 −1·6
−11	+6·5	+ 5·5	−10	− ·6	−3·8	−1·2 −3·2
− 8·5	+8·5	+10	− 5	+1·6	+1·0	−5·4 − ·6
− 8	+5·5	+ 8·5	− 8·5	+ ·1	−1·6	+ ·9 −1·7
− 9	+8	+10·5	− 9·5	+1·1	−1·4	−2·7 −2·5
− 8·5	+4	+ 7	− 8	−1·0	−2·0	− ·4 −1·0
					Mean	−1·62

Calculated reading for equilibrium position 24″·85400.

Reduction of the Observations.

§ 31. Let L = the distance between the electrodes in any equilibrium position.
n = the corresponding rate of rotation (number of revolutions per second) of the disc.
t = the temperature of the mercury in the trough.
θ = the temperature of the tuning-fork.
P = the vibration frequency of the fork at 15°·5 C.
P_θ = the vibration frequency of the fork at temperature θ.
ρ = the specific resistance of mercury at 15°·5 C.
ρ_t = the specific resistance of mercury at temperature t.
A = the sectional area of the mercury column at 15°·5 C.
A_t = the sectional area of the mercury column at temperature t.
k = the temperature coefficient of the tuning-fork vibration frequency [= − ·00011].
α = the temperature coefficient of the specific resistance.
γ = the cubical coefficient of expansion of mercury.
N = the number of teeth in the row on the cylindrical drum viewed through the fork slit.

[The difficulty of introducing a correction for the changes in the trough consequent on the small variations of temperature during the observations has been explained in § 15. In all that follows the dimensions of the trough are assumed to be constant.]

We have

$$Mn = \frac{L\rho_t}{A_t}$$

$$n = \frac{2P_\theta}{N} = \frac{2P(1 + k\overline{\theta - 15\cdot5})}{N}$$

$$\rho_t = \rho_0(1 + \alpha\overline{t - 15\cdot5})$$

$$A_t = A(1 + \gamma\overline{t - 15\cdot5})$$

$$\therefore 2MP\frac{1 + k\overline{\theta - 15\cdot5}}{N} = \frac{L\rho}{A}(1 + \overline{\alpha - \gamma}\,\overline{t - 15\cdot5})$$

or

$$2MP\nu = \frac{L\rho}{A}$$

where

$$\nu = \frac{1 + k\overline{\theta - 15\cdot5} - \overline{\alpha - \gamma}\,\overline{t - 15\cdot5}}{N}.$$

Let ν_1, ν_2 be the values of ν for two equilibrium positions of the movable electrode separated from one another by a distance l, the quantity of mercury in the trough being the same for both positions.

Then

$$2MP\,(\nu_1 - \nu_2) = \frac{l\rho}{A}$$

or

$$2MP\mu = \frac{l\rho}{A}, \text{ where } \mu = \nu_1 - \nu_2$$

or

$$2MPA = s\rho, \text{ where } s = \frac{l}{\mu}\,.$$

In each complete set of observations four equilibrium positions are taken, two with the mercury in the trough at one level, two at another.

Let

s_1, A_1 be the values of s and A with the mercury at the first level,

and

s_2, A_2 their values when new mercury has been added.

Then

$$2MP\,(A_2 - A_1) = (s_2 - s_1)\,\rho.$$

But

$$A_2 - A_1 = b\,(h_2 - h_1),$$

where

b = the mean breadth of the trough over the length used,

and

$h_2 - h_1$ = change of level of the mercury surface, assuming that before and after the addition of fresh mercury the temperature is 15°·5 C.;

therefore we have finally

$$\rho = \frac{2MPb\,(h_2 - h_1)}{s_2 - s_1}\,.$$

The results of calculation are given in Table VI. (p. 40).

In the column marked c the actual readings on the long screw and wheel H (Plate 2, fig. 5) are recorded. The first of the two columns marked l contains the distances passed over by the movable electrode, reckoned by taking the differences of these readings. The second of the columns marked l contains these differences corrected by reference to the standard bars (*vide* § 11).

In the column marked $h_2 - h_1$ are placed the actual differences of height of the two mercury surfaces obtained by simple interpolation from the spherometer calibration table in § 29; in the column next to it these differences corrected for temperature as follows :—

Let σ_1 = the height of the mercury surface above the base of the trough, when the first spherometer reading is taken at temperature t_1,

and σ_2 = the height of the mercury surface above the base of the trough, when the second spherometer reading is taken at temperature t_2.

Then $\sigma_2 - \sigma_1$ = the actual difference of height recorded in the column marked $h_2 - h_1$,

$$\text{and } (h_2 - h_1)_{15\cdot 5} = \sigma_2 - \sigma_1 - \gamma\,(\sigma_2\,\overline{t_2 - 15\cdot 5} - \sigma_1\,\overline{t_1 - 15\cdot 5}).$$

Approximately

$$\sigma_1 = \cdot 75 \text{ in.}$$
$$\sigma_2 = 1\cdot 25 \text{ in.}$$

and these approximate values may be used in calculating the correcting term.

It will be observed that the greatest value of the correction is about 4 in 50,000.

In making the temperature corrections, the following values of the temperature coefficients have been taken :—

$$k = -\cdot 00011\ *$$
$$\lambda_{15\cdot 5} = \cdot 0001798\ \dagger$$
$$\alpha_{15\cdot 5} = \cdot 0009076\ (\textit{vide infra}).$$

Let R_t = the resistance of a column of mercury in a glass tube at $t°$,

R_0 = the resistance of the same at 0°.

Then taking the formula

$$R_t = R_0\,(1 + 0\cdot 0008649\,t + 0\cdot 00000112\,t^2)\ddagger \quad . \quad . \quad . \quad . \quad . \quad . \quad (\epsilon)$$

we have $R_{15\cdot 5} = R_0 \times 1\cdot 013675$.

Let l_t be the length of the glass tube at $t°$, and s_t be the sectional area of the internal channel at the same temperature.

$$\text{Then } R_t = \frac{l_t\rho_t}{A_t} = \frac{l_0\rho_0}{A_0}\,(1 + \overline{\alpha - \beta}\,t) = R_0\,(1 + \overline{\alpha - \beta}\,t),$$

α being the temperature coefficient of the specific resistance,

and β the linear coefficient of expansion of the glass.

Hence $(\alpha - \beta) \times 15\cdot 5 = \cdot 013675$,

and $\alpha \times 15\cdot 5 = \cdot 013799$, taking $\beta = \cdot 000008$.

We have then $\rho_{15\cdot 5} = \rho_0 \times 1\cdot 013799$.

The factor 1·013799 was used in calculating the specific resistance at 0° C. from that at 15°·5 C.

The value of $\alpha_{15\cdot 5}$ above given is obtained from (ϵ), correction being similarly made for the fact that the formula applies to mercury in a glass tube.

* MACLEOD and CLARKE, 'Phil. Trans.,' 1880.

† REGNAULT.

‡ MASCART, NERVILLE and BENOIT, 'Journal de Physique,' 1884, p. 230.

TABLE VI.—The Results of Calculation.

Set No.	N.	t.	θ.	ν.	c.	l. (uncorrected).	l.	μ.	S.	$s_2 - s_1$.	$h_2 - h_1$.	$h_2 - h_1$, corrected to 15°·5 C.	Log. $\frac{h_2 - h_1}{s_2 - s_1}$	Sp. Res. in $\frac{(\text{inch})^2}{\text{sec.}}$ at 15°·5 C.	Sp. Res. in $\frac{(\text{cent.})^2}{\text{sec.}}$ at 15°·5 C.	Sp. Res. in $\frac{(\text{cent.})^2}{\text{sec.}}$ at 0° C.
1	8 13 13 21	15·47 15·48 15·60 15·60	15·10 15·47 15·56 16·15	·1250078 ·0769244 ·0769167 ·0476120	15·1708 24·4341 15·9931 24·8540	9·2632 8·8609	9·2622 8·8588	·0480834 ·0293047	192·629 302·299	109·670	·49672	·49671	$\bar{3}$·6560148	14787·8	95,402	94,103
2	8 13 13 21	15·52 15·51 15·51 15·50	15·70 16·14 15·70 16·20	·1249957 ·0769171 ·0769210 ·0476152	15·2583 24·4866 15·9213 24·8090	9·2282 8·8876	9·2274 8·8854	·0480785 ·0293058	191·924 303·198	111·274	·50385	·50382	$\bar{3}$·6558779	14783·2	95,372	94,074
3	8 13 13 21	15·49 15·51 15·57 15·56	15·69 15·74 14·90 15·30	·1249979 ·0769204 ·0769244 ·0476178	15·1531 24·4225 15·9294 24·8143	9·2694 8·8849	9·2683 8·8826	·0480775 ·0293065	192·778 303·092	110·314	·49961	·49957	$\bar{3}$·6559683	14786·3	95,392	94,093
4	8 13 13 21	15·64 15·64 15·36 15·39	15·87 16·19 15·54 15·54	·1249823 ·0769091 ·0769303 ·0476226	15·1278 24·4041 15·9434 24·8214	9·2764 8·8780	9·2754 8·8757	·0480732 ·0293077	192·943 302·845	109·902	·49748	·49745	$\bar{3}$·6557453	14778·7	95,343	94,045
5	8 13 13 21	15·19 15·17 15·56 15·57	15·85 15·70 16·44 16·70	·1250232 ·0769396 ·0769115 ·0476102	15·2458 24·4737 15·9553 24·8261	9·2279 8·8708	9·2270 8·8678	·0480836 ·0293013	191·894 302·643	110·749	·50117	·50116	$\bar{3}$·6556345	14774·9	95,318	94,021
													Mean	14782·2	95,365	94,067

§ 32. The final result arrived at is that the specific resistance of mercury at 15°·5 C. is equal to

$$14782{\cdot}2 \frac{(\text{inch})^2}{(\text{sec.})} = 95365 \frac{(\text{centimetre})^2}{(\text{sec.})},$$

and hence that the specific resistance of mercury at 0° C. is equal to

$$94067 \frac{(\text{centimetre})^2}{(\text{sec.})}.$$

The probable error calculated from the five values given by the individual sets of observations is ± 10.

It follows that an ohm is equal to the resistance of a column of mercury of 1 square mm. sectional area, and 106·307 centimetres long.

§ 33. In conclusion it may be useful to consider the improvements that might be introduced in designing new apparatus for a further and more accurate determination of the specific resistance of mercury by this method.

Attention may, in the first place, be directed to the brushes in order to obtain steadiness of the galvanometer needle when the galvanometer circuit is complete, the battery circuit broken, and the disc in rotation. The perforated brush fed by a constant stream of mercury from a cistern of adjustable height goes far to secure this when the disc has been carefully ground true in place; and if, instead of one such brush, three or four were placed at different points of the circumference of the disc, as suggested by ROWLAND, the result would probably be sufficiently good.

In determining the equilibrium positions the variation in W—E (*vide* § 22) corresponding to a given displacement of the movable electrode is proportional to the current through the standard coil and the mercury trough; and the accuracy with which the equilibrium position can be determined may therefore be increased by increasing the current used. But there are two considerations which limit the magnitude of the current, viz., first and mainly, the unsteadiness of the galvanometer needle due to small variations in the rate of rotation of the disc, an unsteadiness which increases proportionately when the current is increased; and secondly, the heating of the wire of the standard coil. The heating of the wire of the coil may be diminished by increasing its sectional area; and there is no objection to this provided the coil is large enough to allow a sufficient number of turns in a single layer to produce an induction coefficient of sufficient magnitude. The main point needing attention, if a large current is to be advantageously used, is the rate of rotation of the disc. The more constant this can be made, the greater will be the current that can be employed, and therefore the more accurate will be the determination of the equilibrium position. In designing new apparatus no effort should be spared to compass uniformity in this respect. The variations in speed depend on variations in the lubrication of the bearings, and in the friction between the brushes of the electromotor and the com-

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mutator, and in less degree perhaps in the friction between the brush or brushes at the disc and the disc circumference. A forced system of lubrication at all the bearings would probably effect a great improvement; the brushes should press as lightly as possible on the commutator of the electromotor, which ought to be newly and very carefully turned; and a heavy fly-wheel should also be used.

In regard to the standard coil, the silk covering of the wire introduces difficulty into the measurement of the mean radius, owing to the compressibility of the covering and its varying thickness. It would be preferable to have a coil of naked wire wound in a screw thread cut on a coil cylinder made of insulating material. But it does not seem easy to find an insulating material that will retain its shape when made into a cylinder of such large dimensions. Wood naturally suggests itself, but it would be difficult to prevent it from altering from time to time with varying atmospheric conditions. Possibly a coil cylinder of wood, built up of a great many pieces, and supported on both sides by heavy phosphor bronze plates, might give satisfactory results. If marble, after a prolonged immersion in molten paraffin wax, is a sufficiently good insulator, it might be tried. There is, however, always the alternative of using covered wire on a metal cylinder of such dimensions as to make the errors of measurement due to the silk covering of the wire insignificant to the degree of approximation required.

The trough, as has been already indicated (§ 6), may probably be more successfully made of plate glass or marble. There seems no reason why its dimensions should not be much more considerable than those of the trough used in the experiments described, in which case their accurate measurement would be facilitated. The sectional area of the mercury column may be as great as seems good, provided the length of the trough is sufficient to make the equipotential surfaces plane in the part of the trough which the movable electrode traverses. Further, this part of the trough is the only part that need be prepared with the utmost accuracy. It might, therefore, be well to make the trough in three parts, the middle portion being prepared with its base and sides carefully made plane, while the end portions need not receive the same attentive workmanship. With a large trough three or four feet might be traversed by the movable electrode, instead of some ten inches as in the present experiments.

In any final determination of the specific resistance to one part in ten thousand, an endeavour will doubtless be made to estimate it still more closely, and to obtain a decimal place further as a mean of a number of determinations. In any such attempt great care will have to be taken to maintain all parts of the apparatus (except the motor, fly-wheel, and connecting shafting) at definite and ascertainable temperatures, in a room specially designed for that purpose.

The author desires to acknowledge the great help he has received at all stages of these experiments from his assistant, Mr. S. T. HARRISON; and to thank Mr. J. GAVEY, of the South Wales Postal Telegraphic Department, for kindly placing at his disposal the telegraphic apparatus required.

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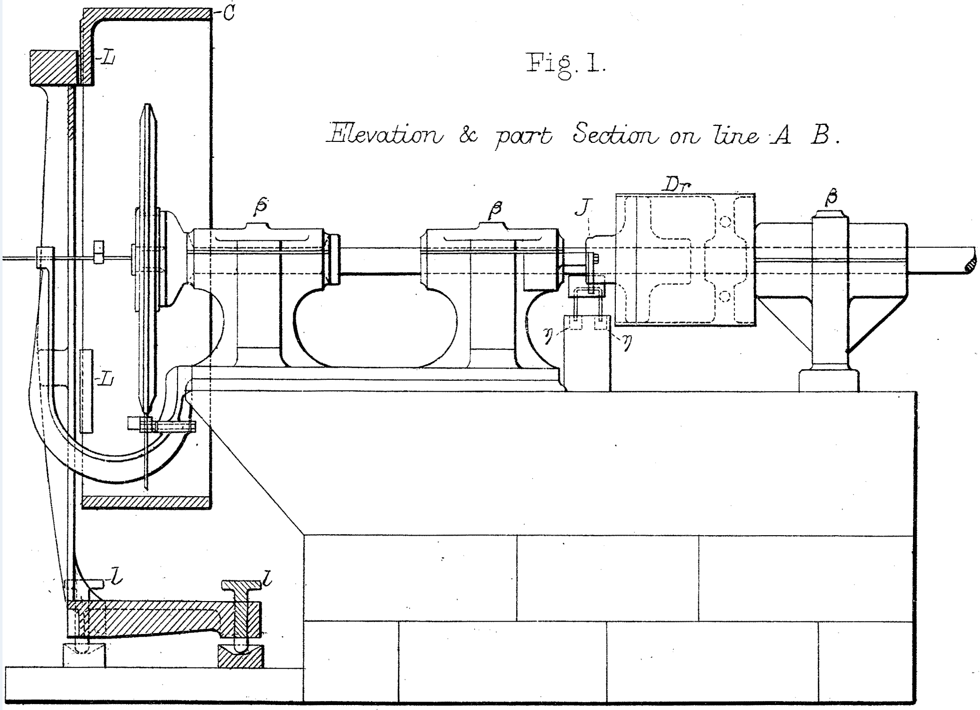
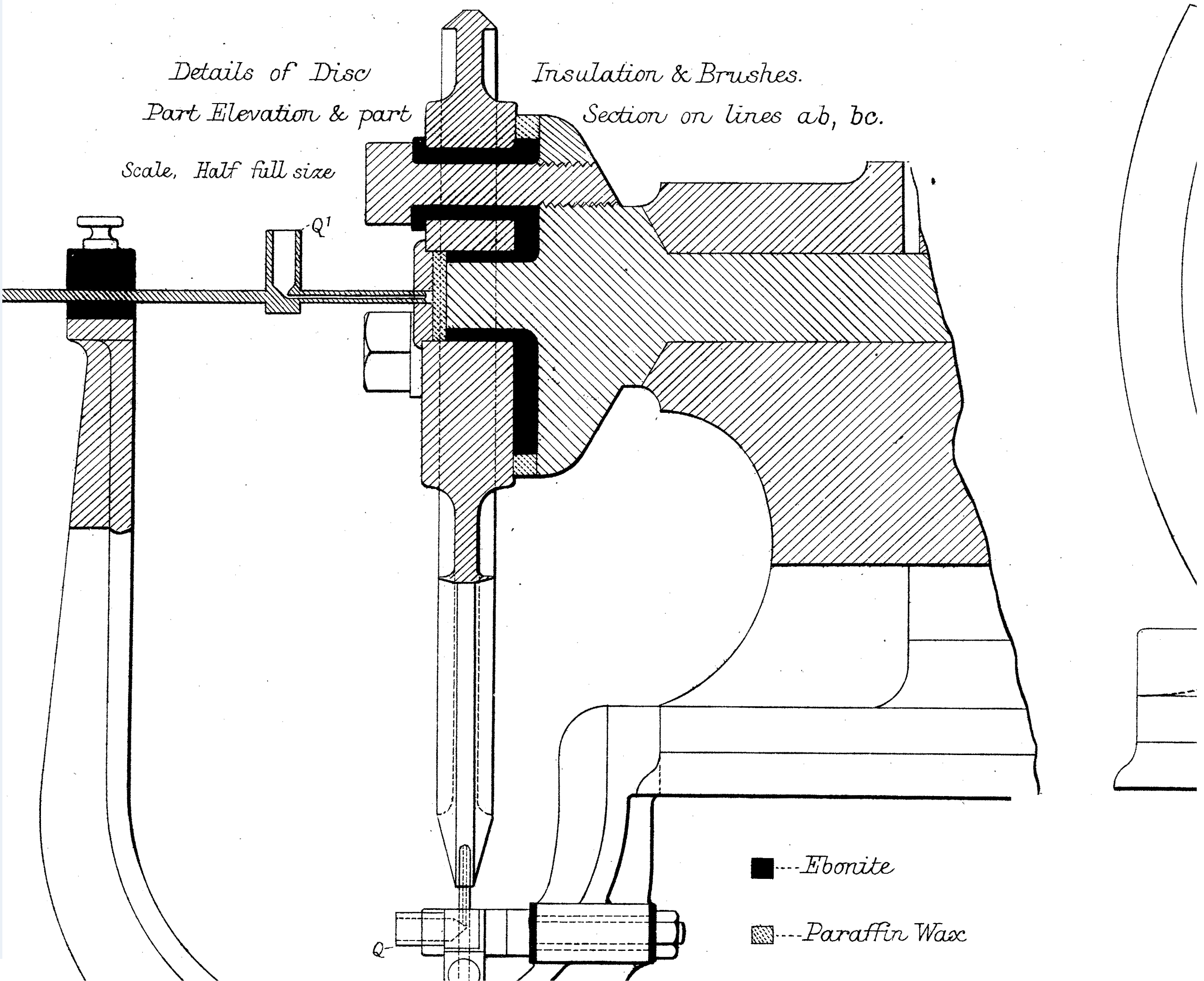
Disc and Coil.
Scale, 1/8th Full size.

Fig. 1.

Elevation & part Section on line A B.



Details of Disc Insulation & Brushes.
Part Elevation & part Section on lines ab, bc.
Scale, Half full size



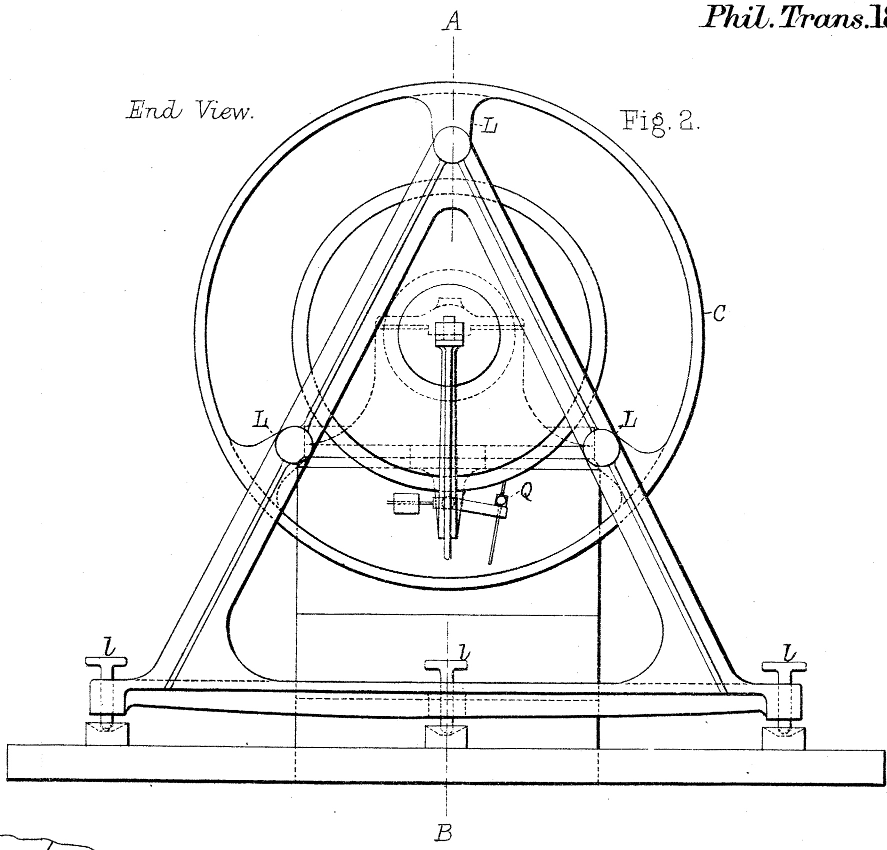
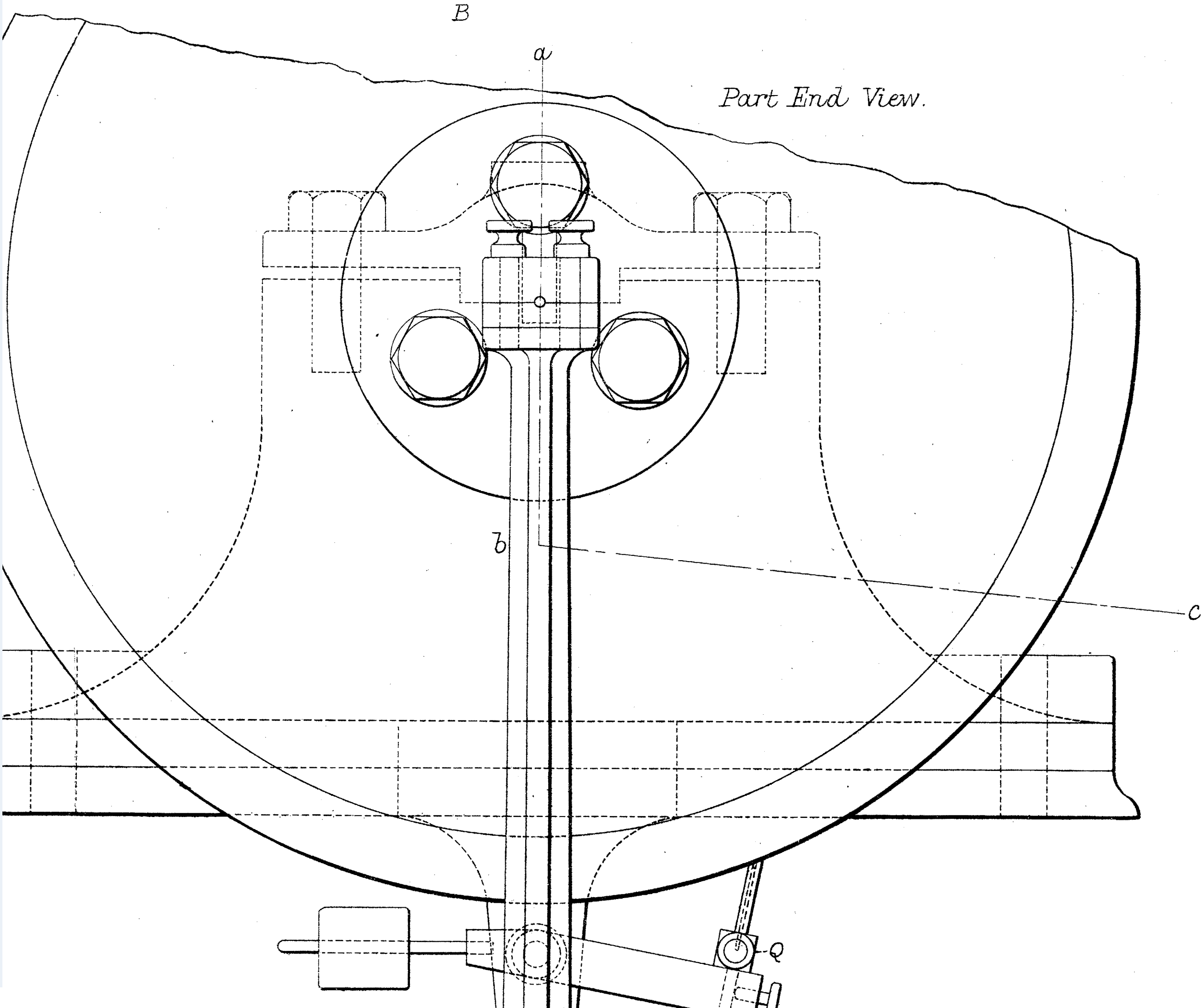


Fig. 2.



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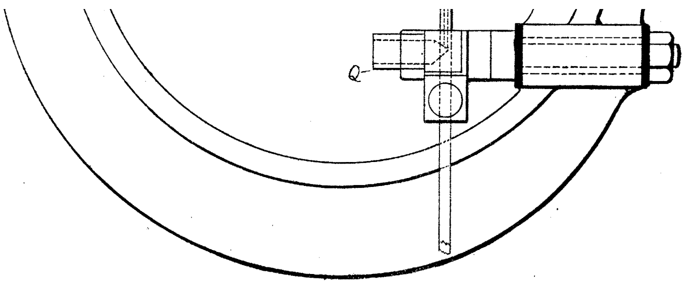

Fig. 3.

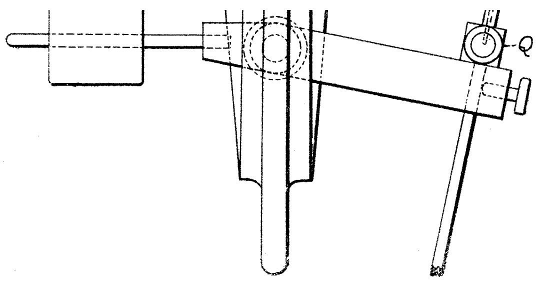

Fig. 4.

West, Newman lith.

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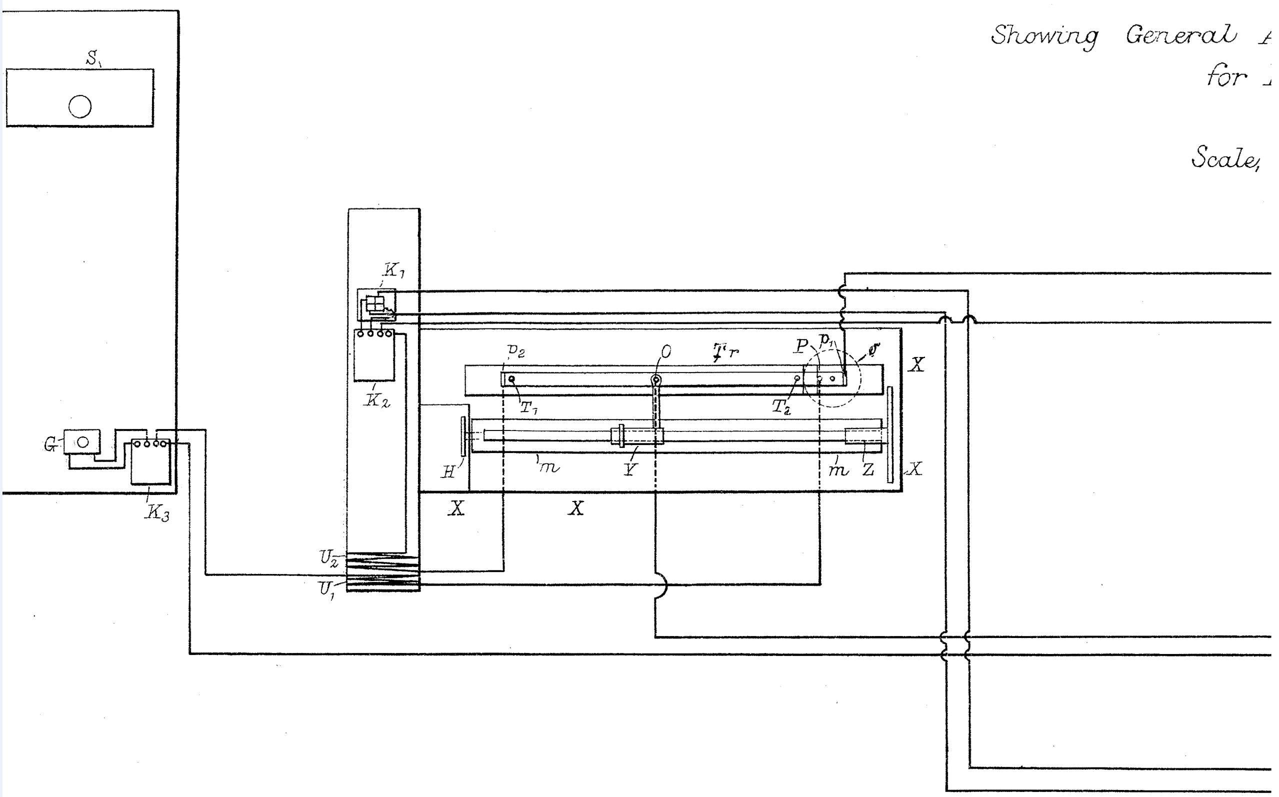
Showing General A
for
Scale,



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Scal

Fig. 6. *Elevation.*

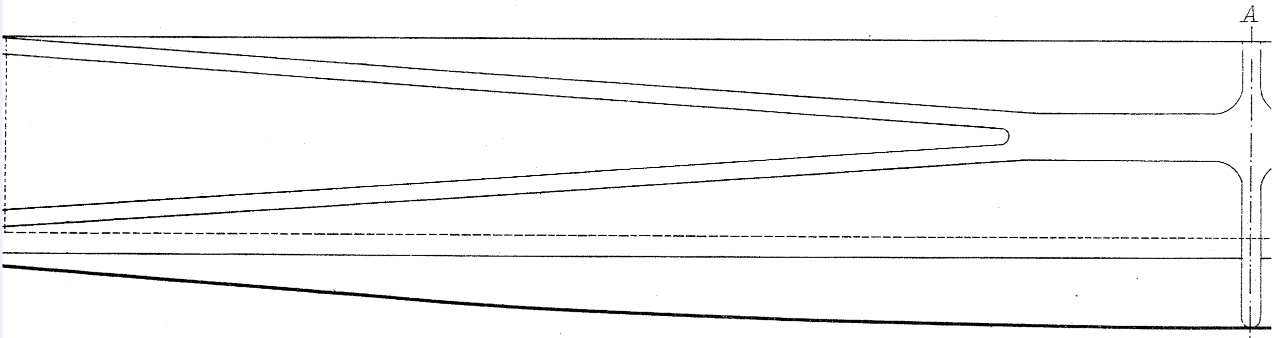
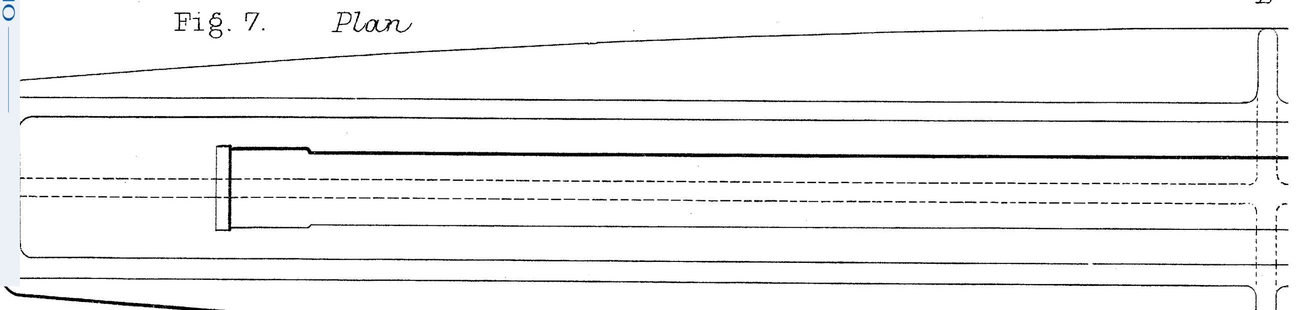


Fig. 7. *Plan*



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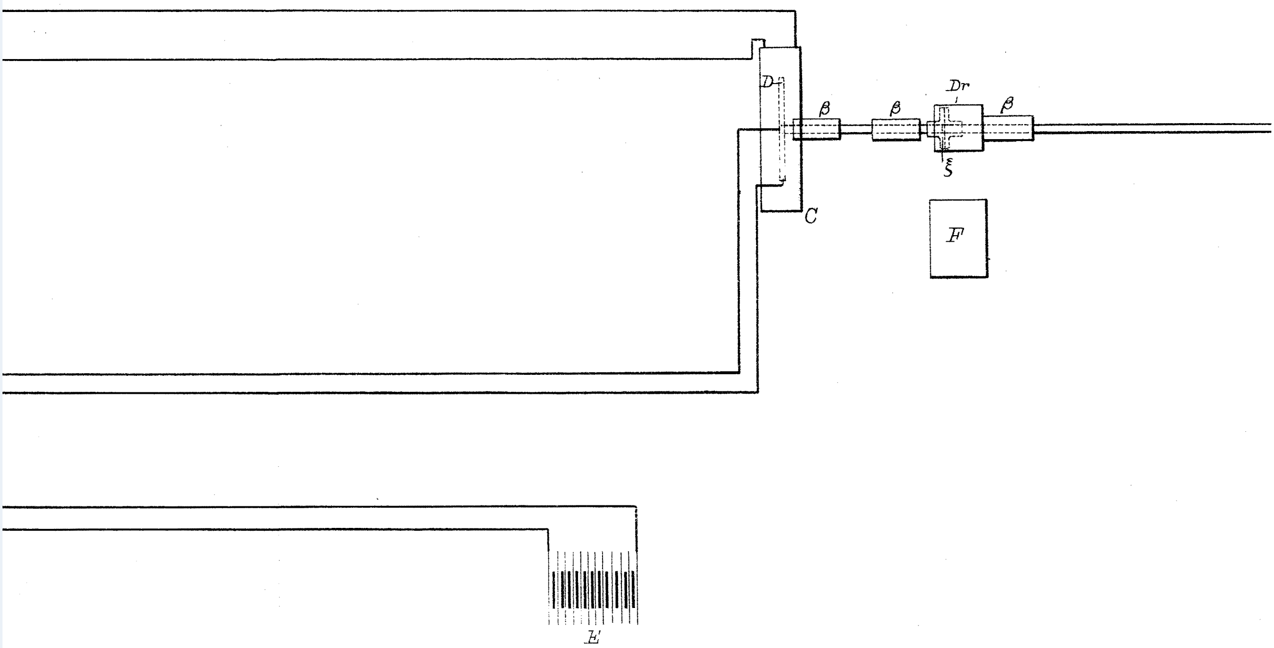
Plan

Arrangement of Apparatus.

Experiment.

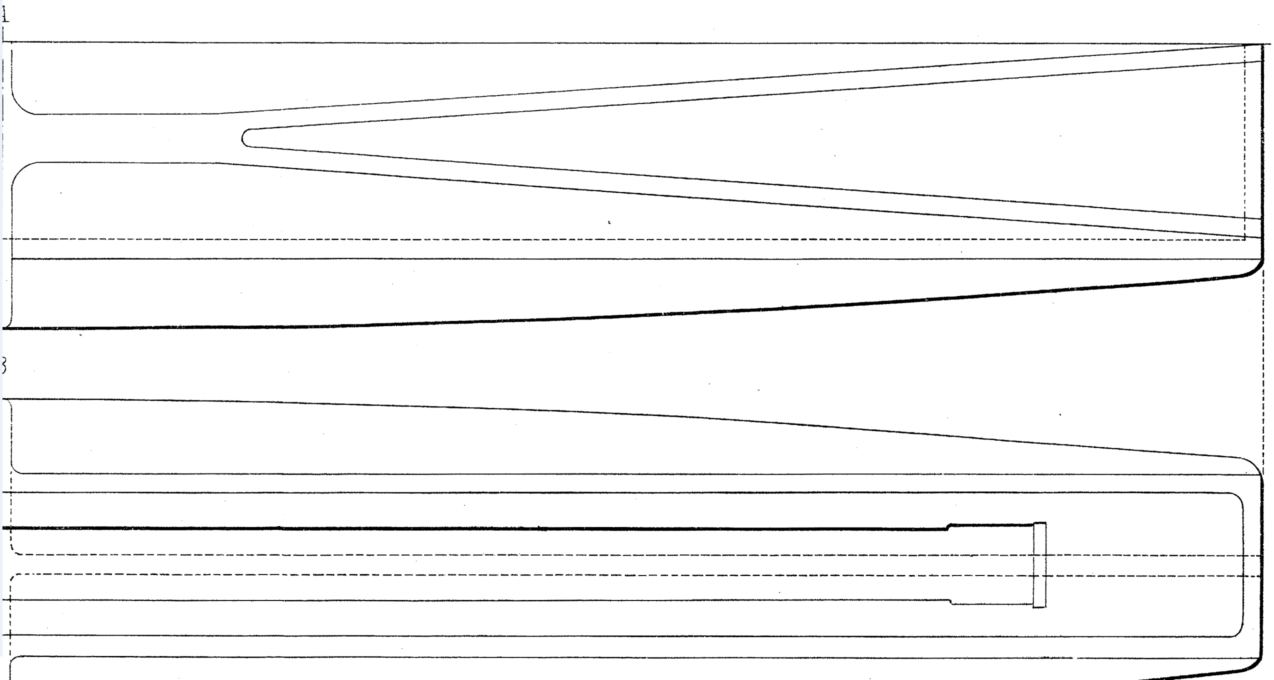
, ½in. = 12ins.

Fig. 5.



rcury Trough.

le, ¼ Full Size.



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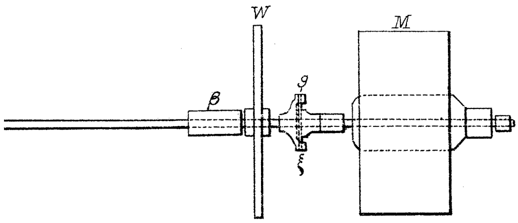
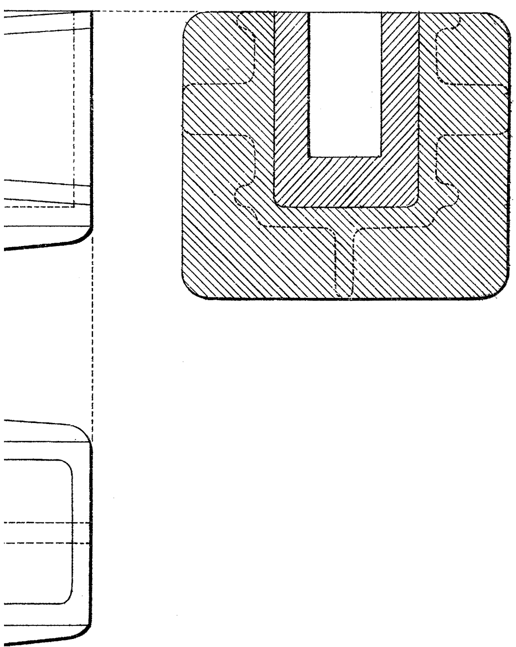
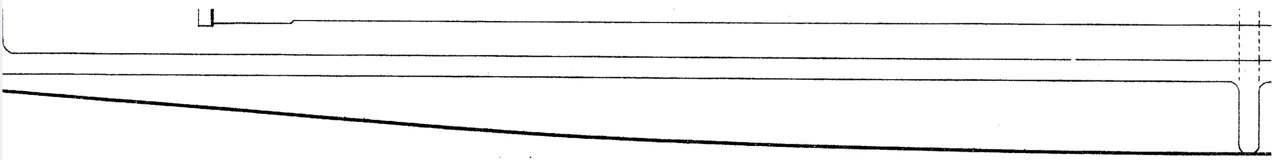


Fig. 8.

Cross Section on line A.B.





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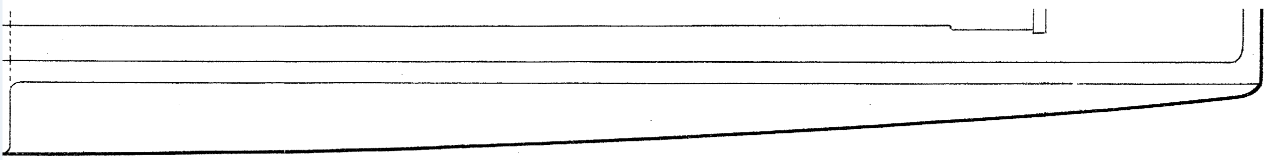

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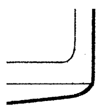

West, Newman lith.

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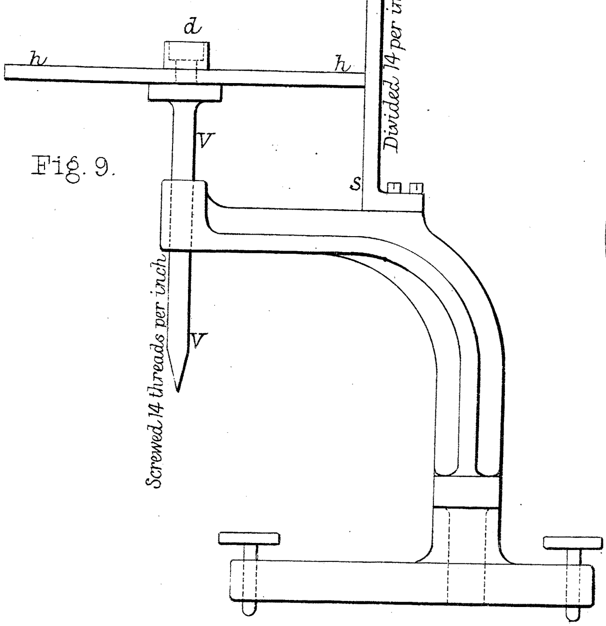
Spherometer.

Scale, 1/4 Full size.

Elevation.

Fig. 9.



Plan.

Divided into 360 divisions.

Fig. 10.

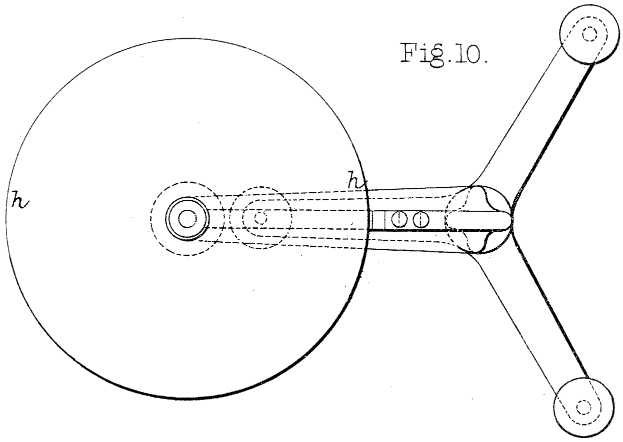
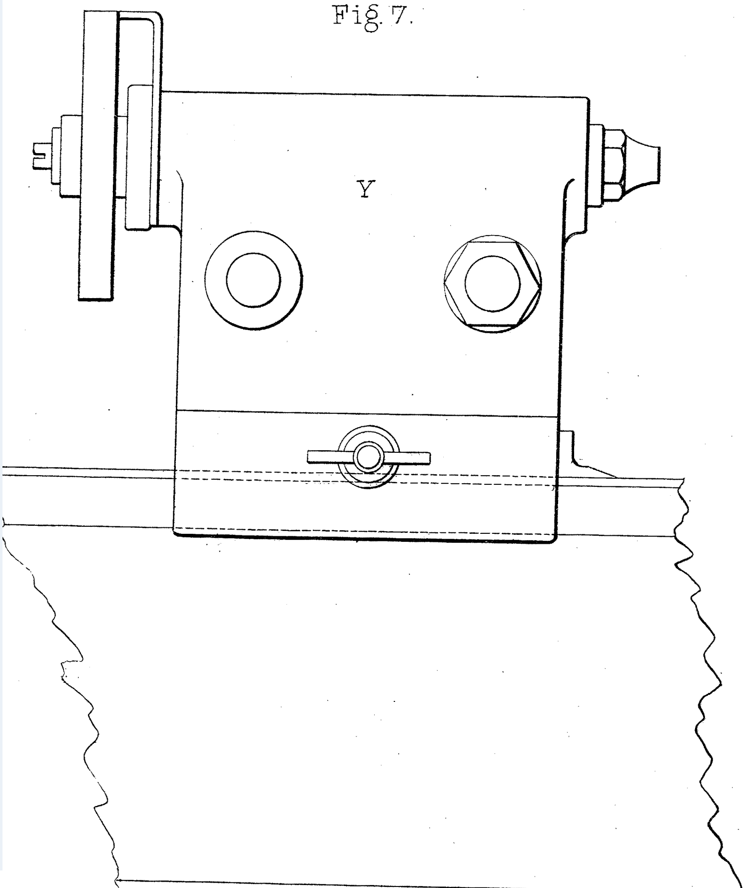
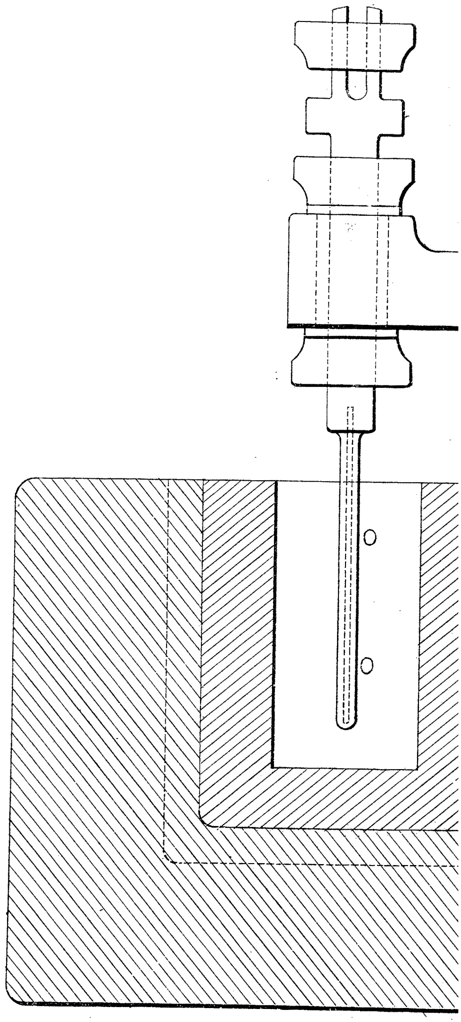


Fig. 7.



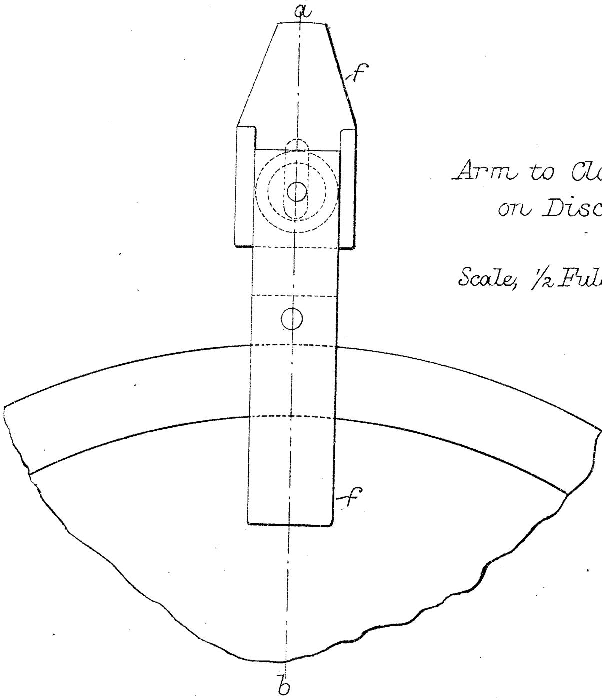


Arm to Clamp on Disc.

Scale, ½ Full size.

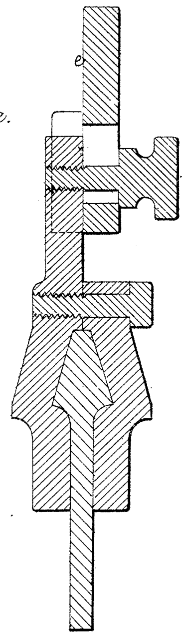
Elevation.

Fig. 13.



Cross Section on line a. b.

Fig. 14.



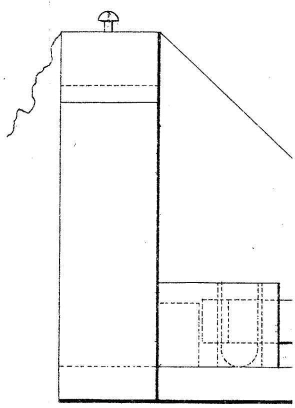
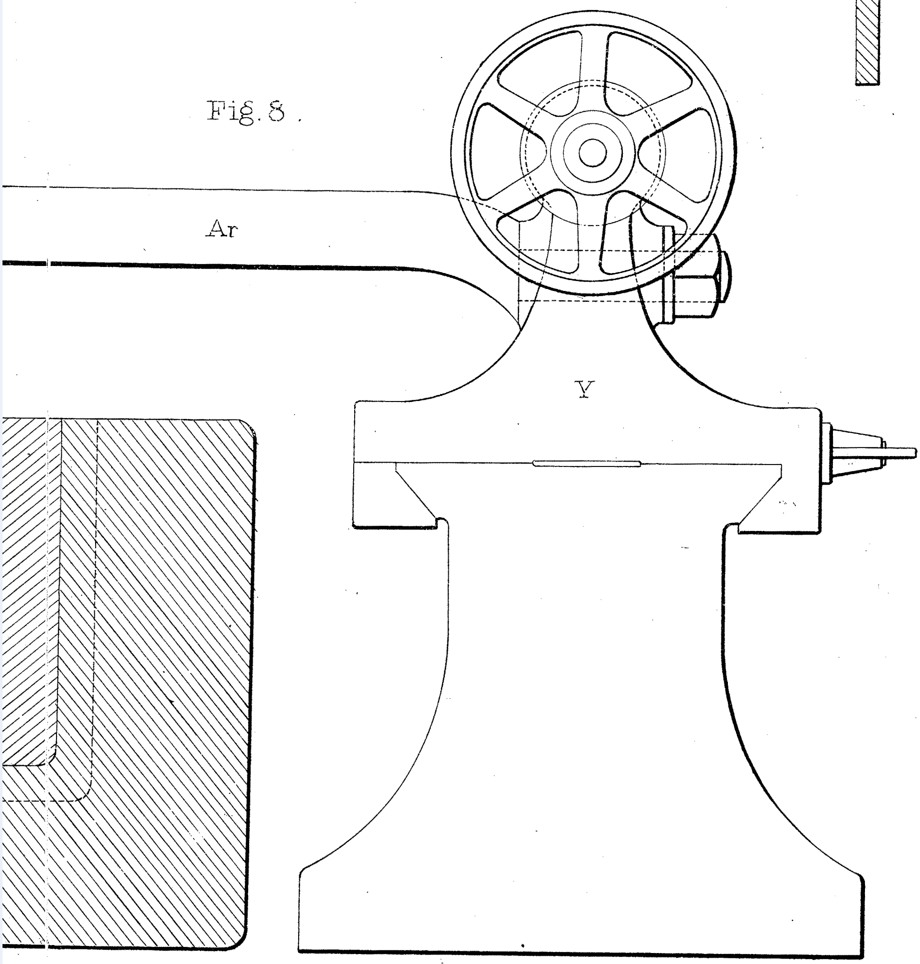
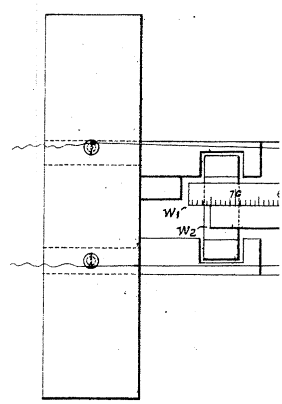


Fig. 8.





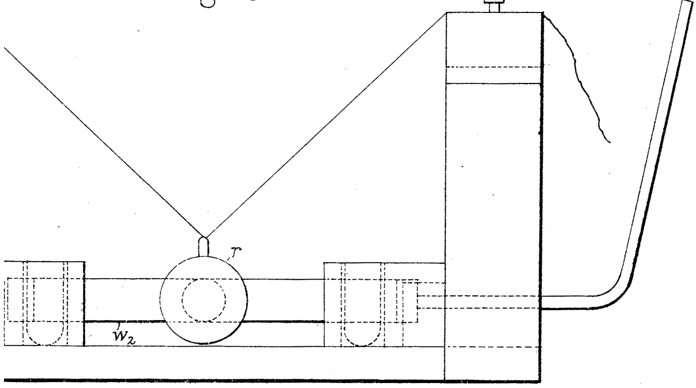
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Callipers.

Scale, ½ Full size.

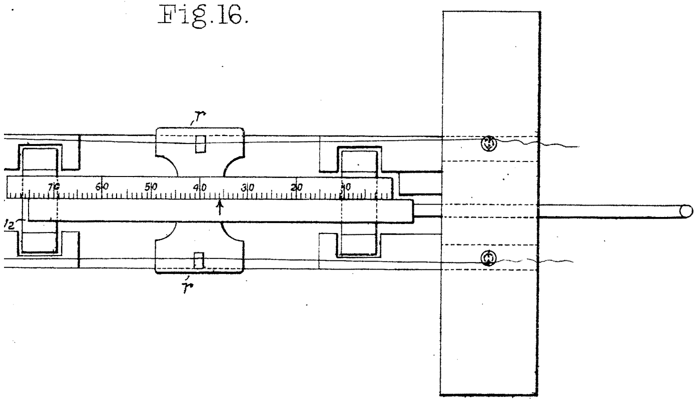
Elevation.

Fig. 15.

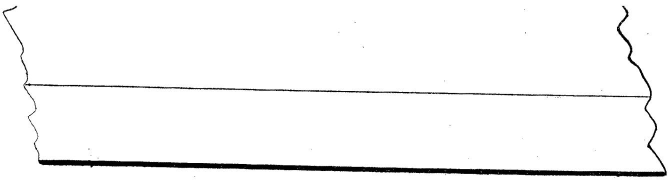


Plan.

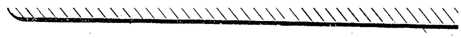
Fig. 16.



MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF



Side View



En

Movable He

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

End View with Trough in Section.

Headstock of Whitworth Machine and Arm carrying Movable Electrode.

Scale, ½ Full Size.

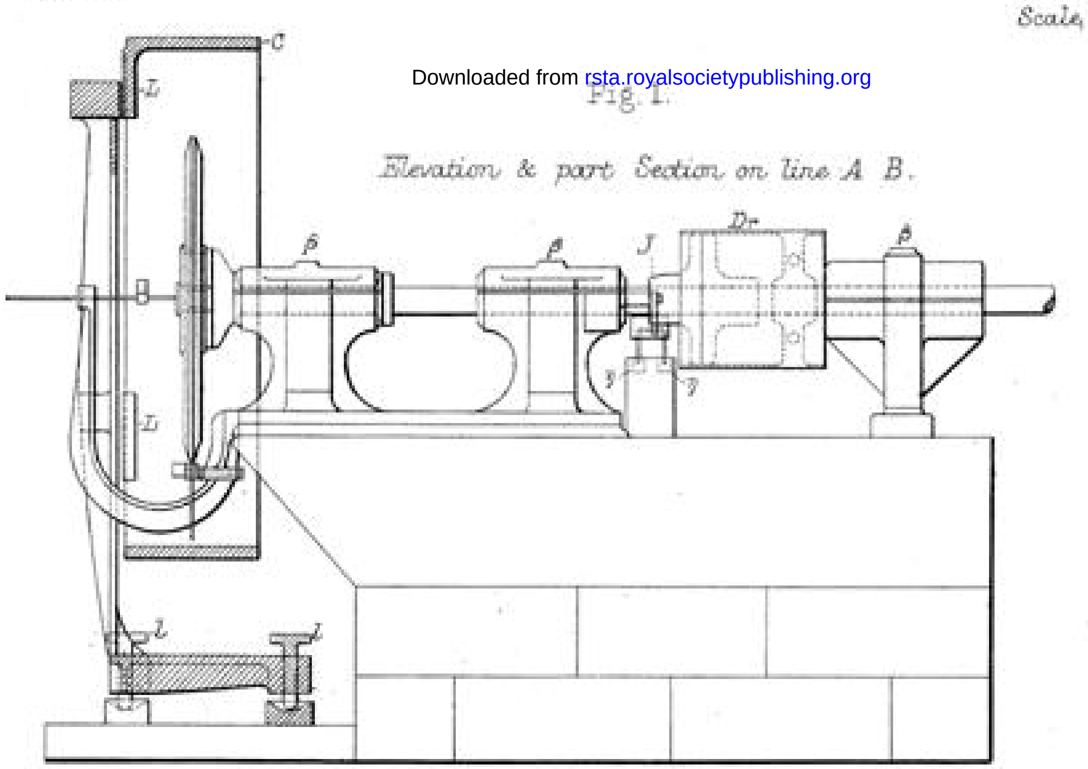
PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

West, Newman lith.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Fig. 1.

Elevation & part Section on line A B.



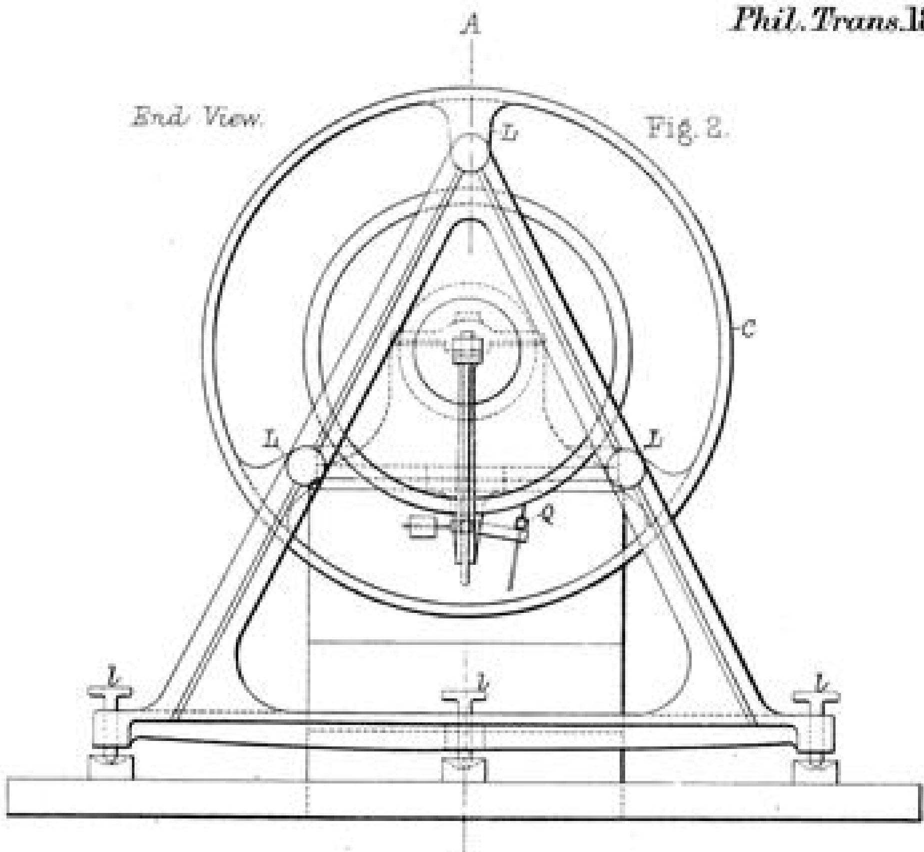


Fig. 2.

Details of Disc

Part Elevation & part

Scale, Half full size

Insulation & Brushes.

Section on lines a.b, bc.

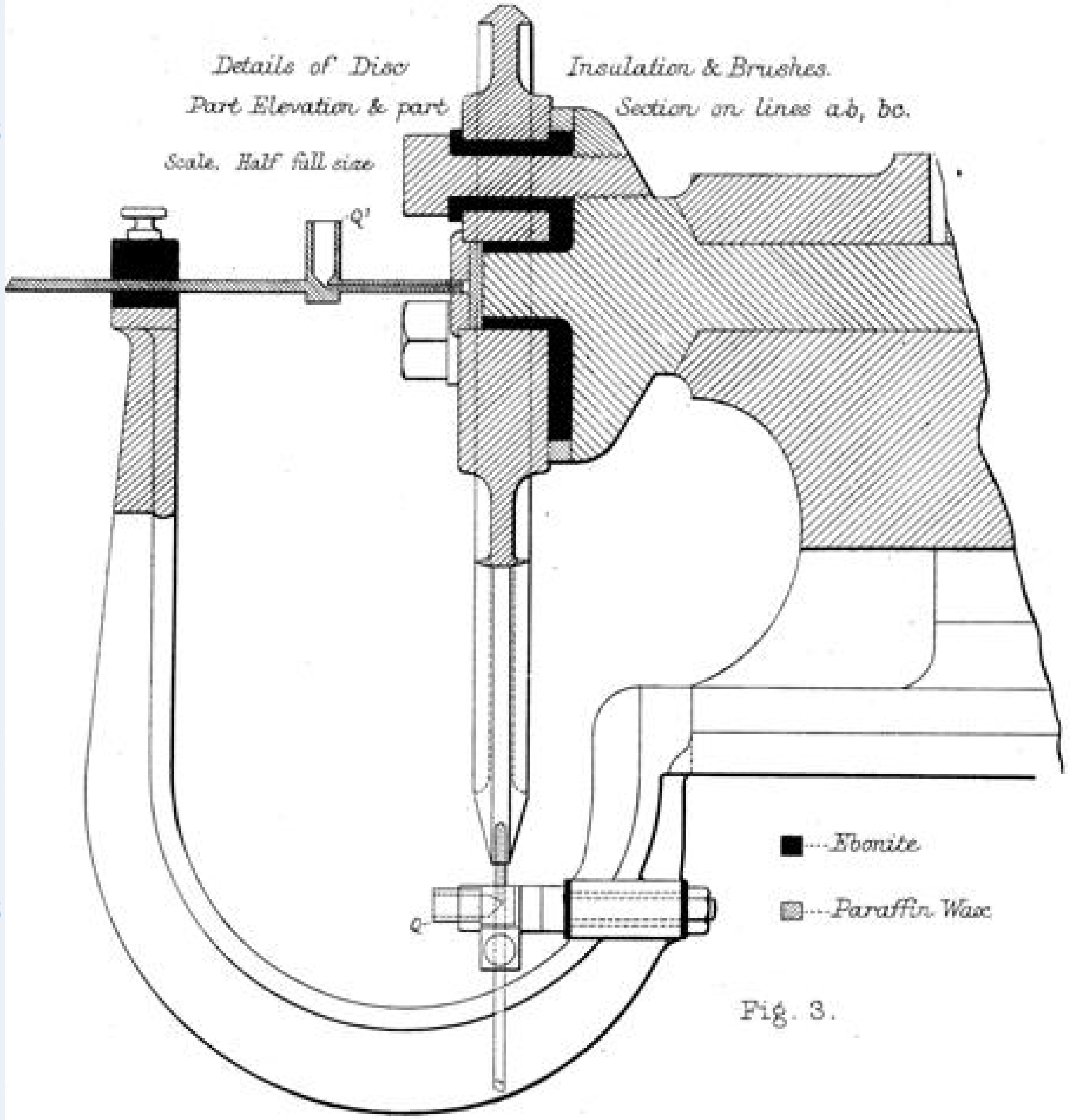


Fig. 3.

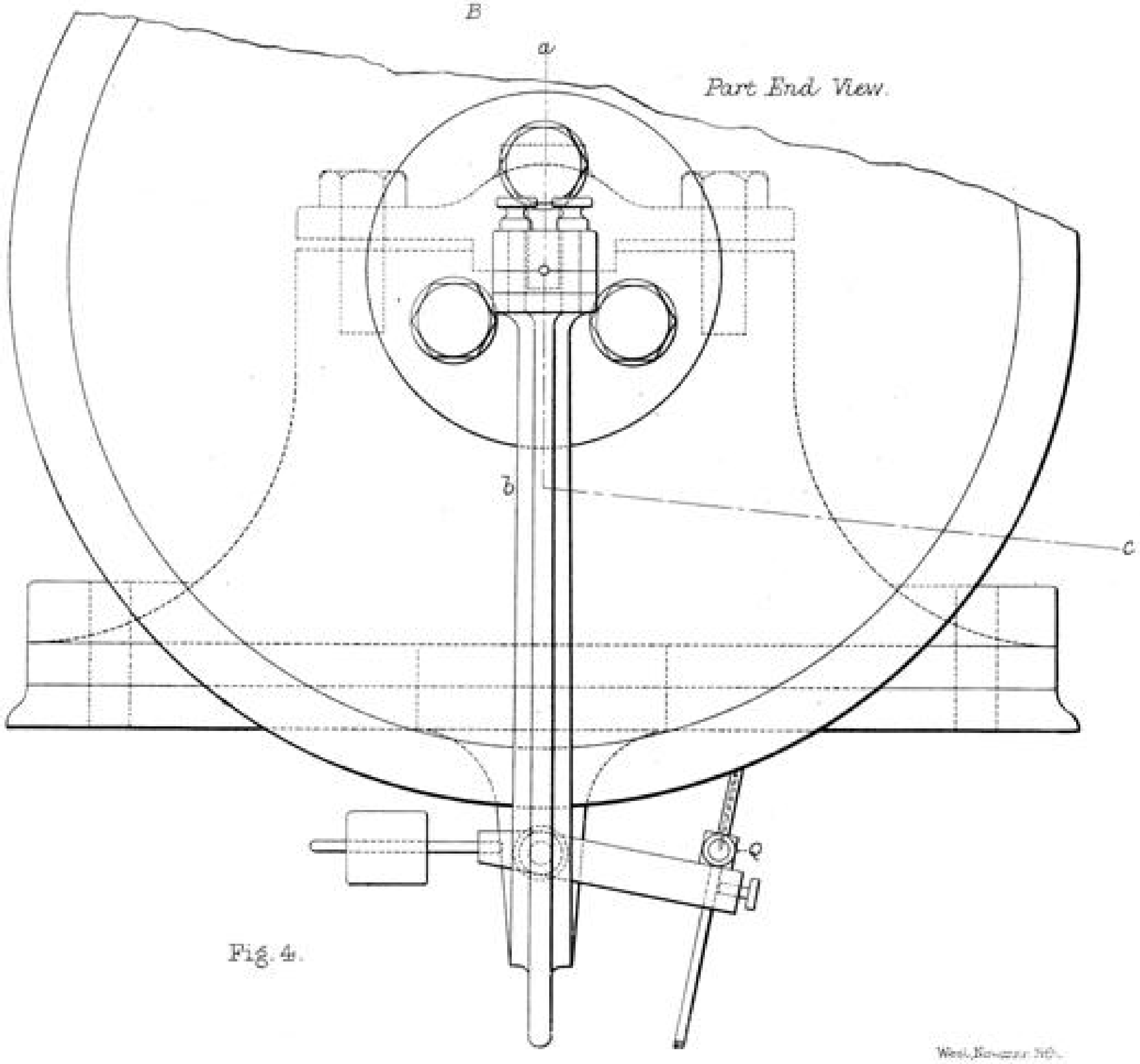
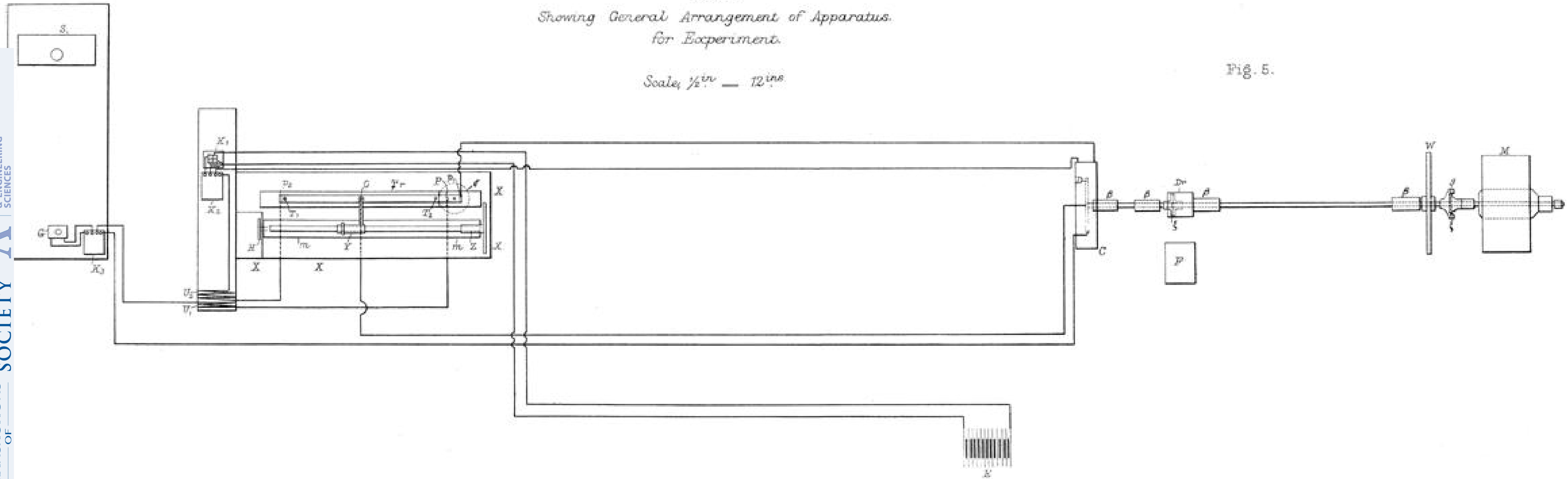


Fig. 4.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Plan
Showing General Arrangement of Apparatus for Experiment.
Scale, ½in. = 12ins.

Fig. 5.



Mercury Trough.
Scale, ¼ Full Size.

Fig. 6. *Elevation.*

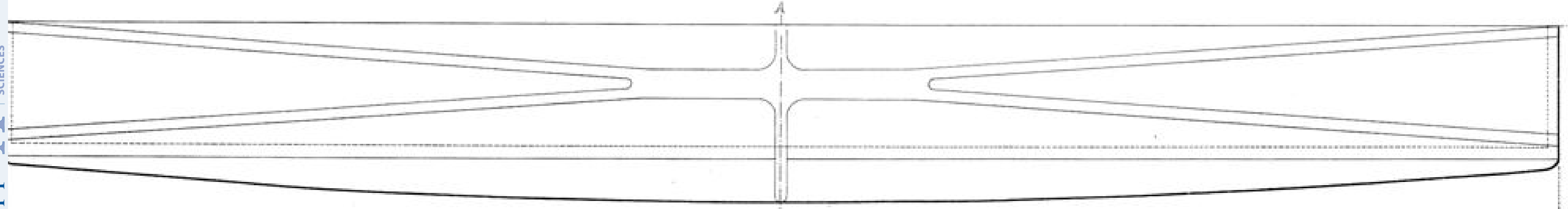


Fig. 8.
Cross Section on line A.B.

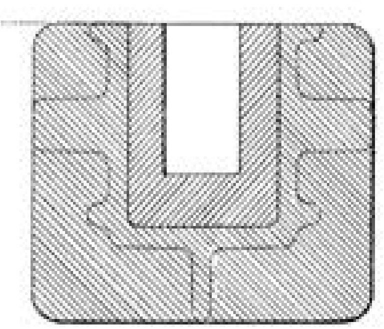
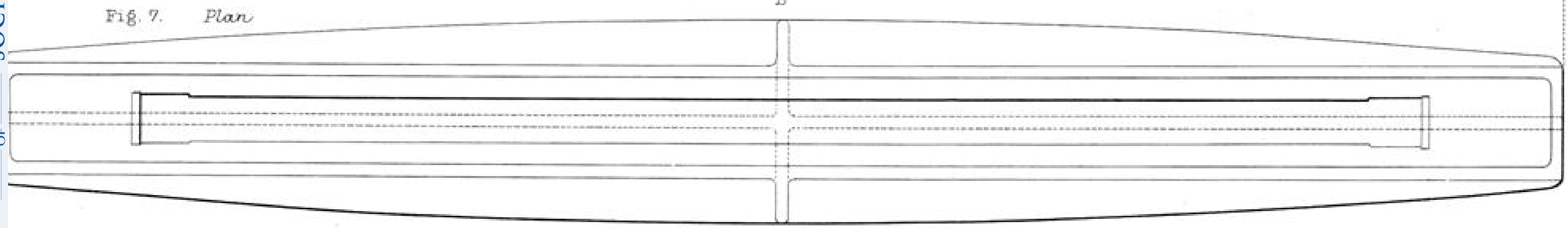


Fig. 7. *Plan.*



West, Newman lith.

Spherometer.
Scale, ¼ Full size.
Elevation.

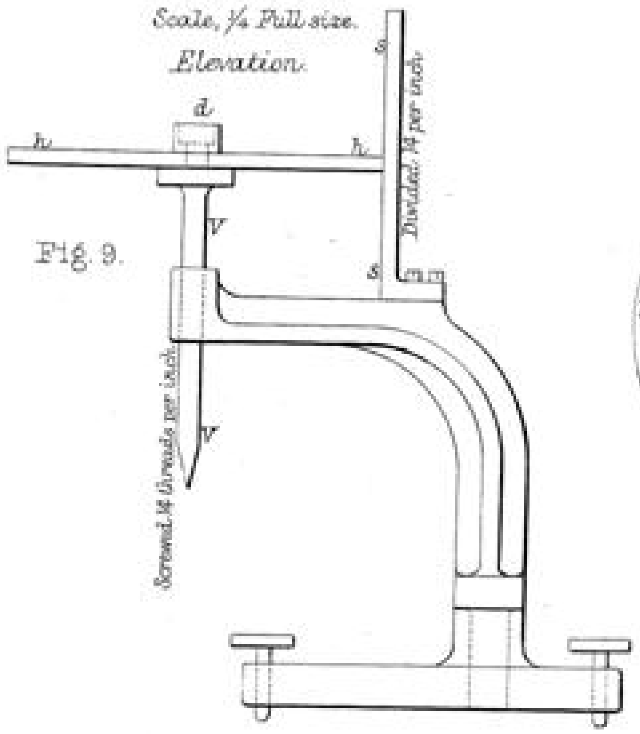


Fig. 9.

Plan.
Divided into 360 divisions.

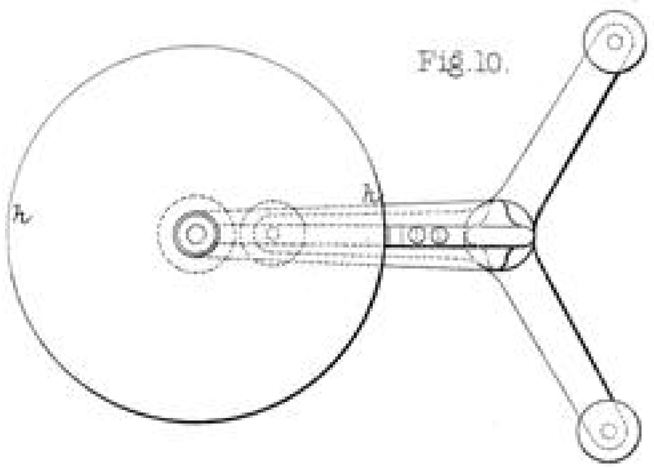
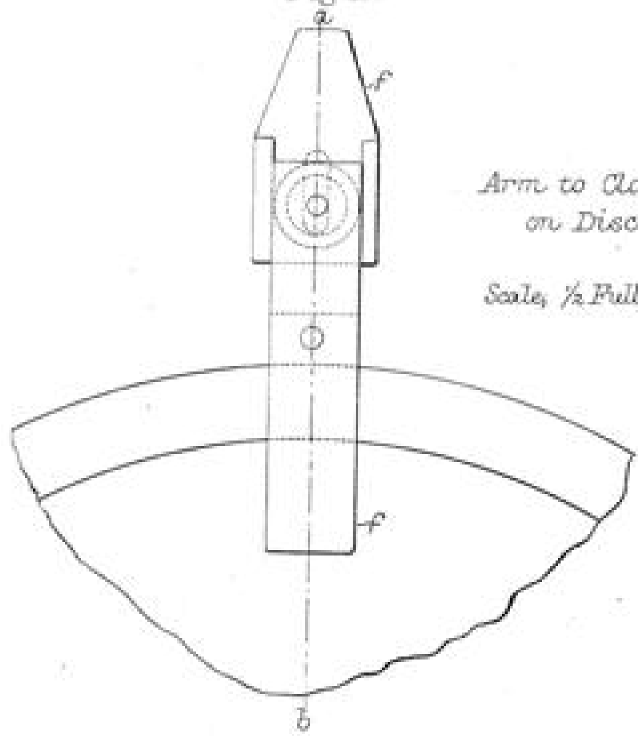


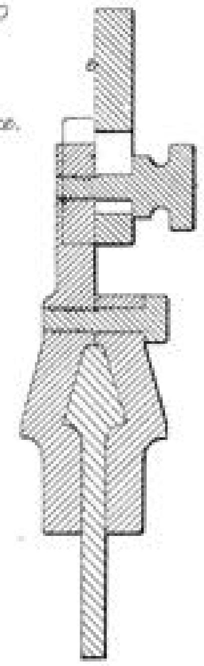
Fig. 10.

Elevation.
Fig. 13.



Arm to Clamp on Disc.
Scale, ½ Full size.

Cross Section on line a. b.
Fig. 14.



Callipers.
Scale, ½ Full size.
Elevation.
Fig. 15.

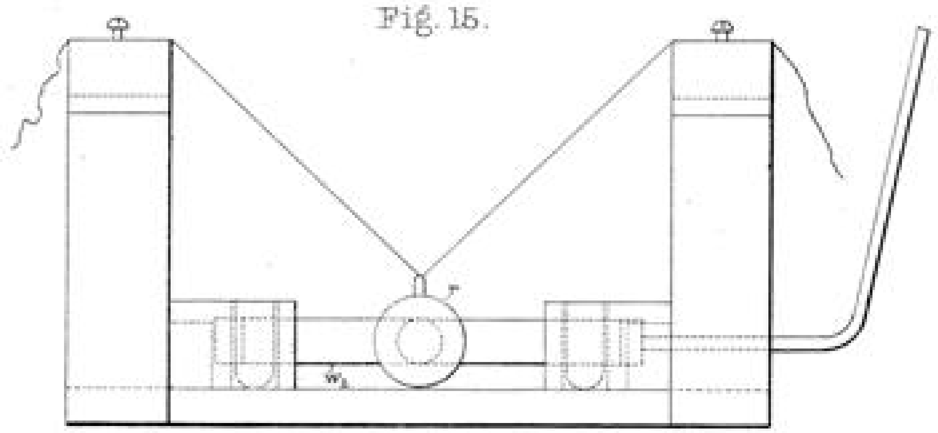
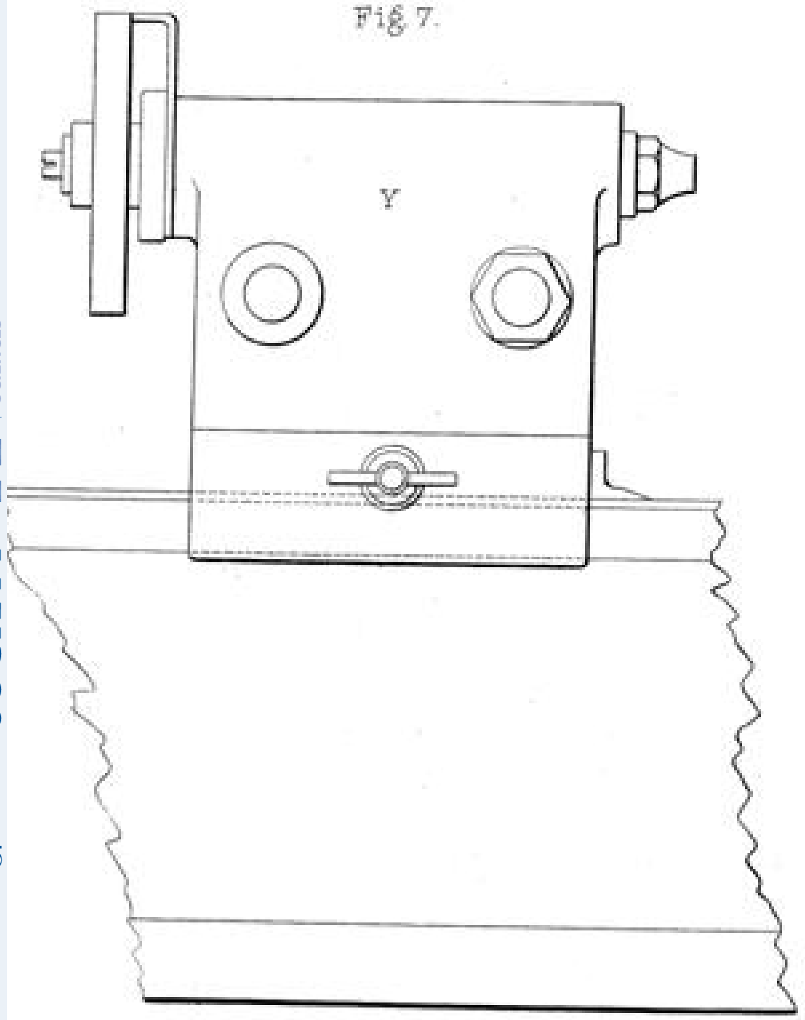
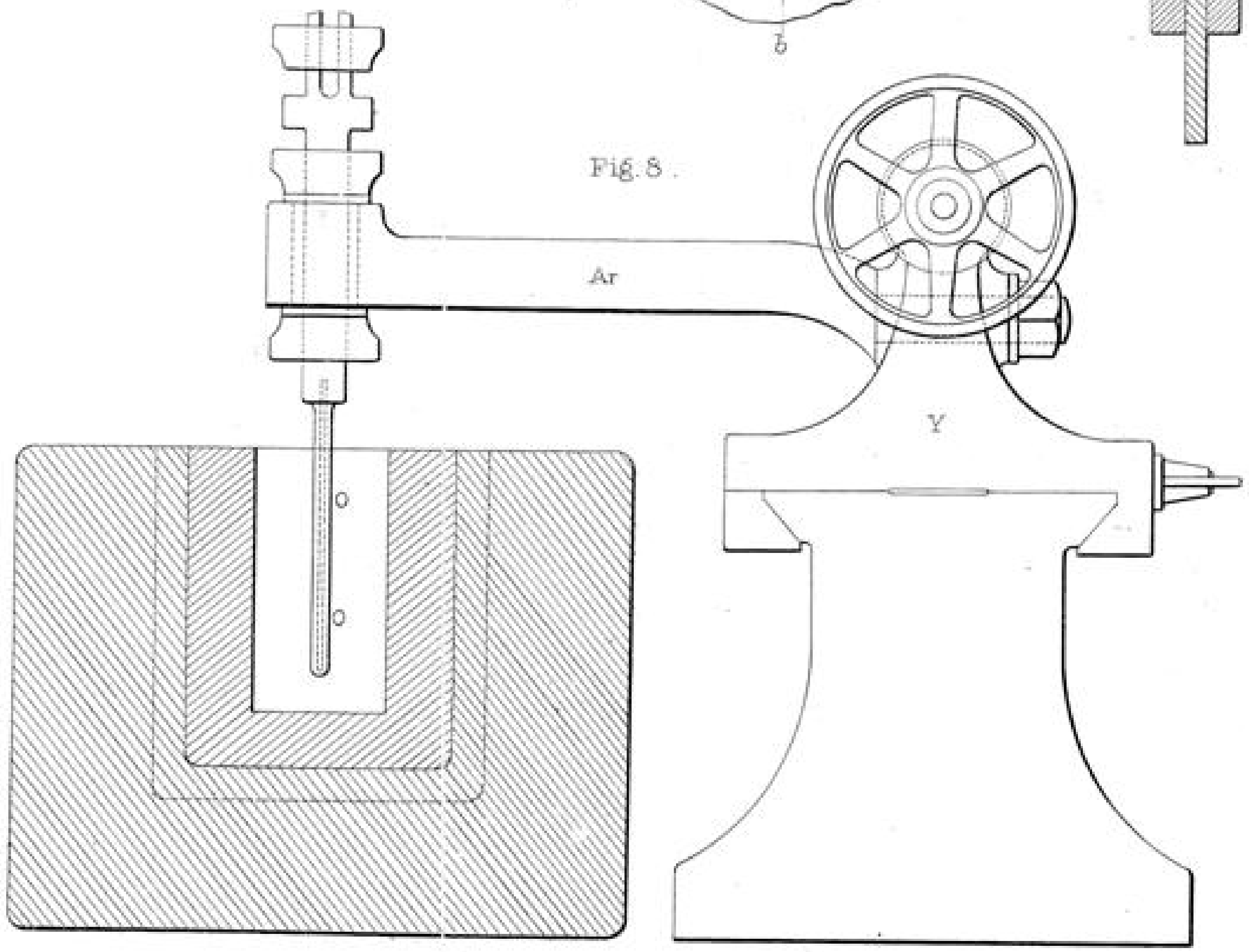


Fig. 7.



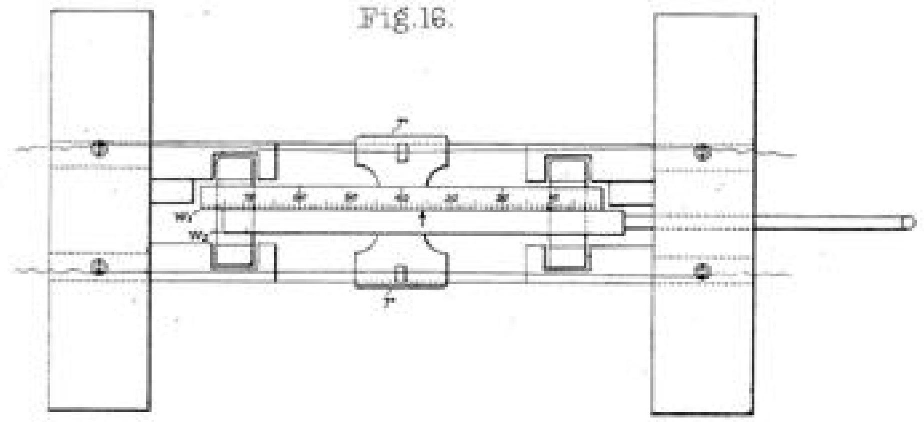
Side View

Fig. 8.



End View with Trough in Section.
Movable Headstock of Whitworth Machine and Arm carrying Movable Electrode.
Scale, ½ Full Size.

Plan.
Fig. 16.



West, Newman lith.